SEPARATUM

ACTA HISTORIAE ARTIUM

ACADEMIAE SCIENTIARUM HUNGARICAE

TOMUS XXV

FASCICULI 3—4

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SCALES AND PROPORTIONS ON DORIC BUILDINGS

BUDAPEST

1979

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by

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Examining the measurements and scales of Greek temples belongs to the very much disputed subjects in the research of European antiquity, both from historical and from aesthetical points of view. Only unprejudiced research can give an answer to such complex and much discussed questions as, for example, on what principles the buildings were planned and what is the role of precision (the calculatedness of ratios) in their aesthetic effect even today.

It is due mainly to factual reasons that, so far, no uniform opinion has been formed on the planning principles of architects living two and a half thousand years ago. Only few of the temples have survived, and what is more, most of them seriously damaged. That is to say, we have to proceed from practice, necessarily inaccurate, to the principles of artistic-architectural design.

These principles are but little known from antique literature. Our first source in time, the manual *De architectura* by Vitruvius, completed probably in 14 B. C., is much more normative in its attitude than would be useful for getting acquainted with the Greek architecture of the archaic and classic periods. Moreover the masters of the Hellenistic age, whom he considered the models to be followed, were esentially different in their views from the artistic practice between the 6th and 4th centuries B. C.

The actual difficulties were further aggravated by occasionally direct ideological prejudices that have deformed the faithful and exact study of the antiquity by dogmatic or seemingly modern, but in fact erroneous interpretations. Among them are e.g. the uncritical idealization of Vitruvius, projecting mystical numerology back to Greek architecture (explaining fronts and ground plans by drawing a pentagon, a hexagon or a decagon in a circle), the unhistorical use of trigonometric calculations, the attitude to explain mathematical inaccuracy by "artists' or art's irrationalism", etc.

Research based on respect for facts and aiming at the most precise reconstruction and arrangement of data, had its beginnings about one and a half centuries ago. John Pennethorne called attention to the horizontal curvature in 1837; a year later Hofer's and Schaubert's publications were issued; and it was in 1851 that F. C. Penrose prepared the list of measurements of the Acropolis and of some other buildings.

Publications on archeology printed since that time are so numerous that it is really impossible to sum them up.

Data of several newly discovered buildings have been added to the actual ones, i.e. to the basis of any generalization; and at the same time data once considered to be indisputable have been revised and altered. Despite factual differences, tendencies in the interpretation of the planning principles of Greek temples can be clearly distinguished.

1. The view considering ancient units of measurement (foot, ell) to be the units of planning.

2. The mathematical interpretation referring to Vitruvius, considers the module a planning principle without any regard to practical units of measurement. That is, it traces back netted structure — planning by small, geometrical quadrangles (mainly squares) and a tendency demanding mathematical accuracy — as far as the 5th century B. C.

3. The geometrical view that attributes planning on the basis of circumcircuses to the Greeks, a conception which, by taking pentagons and decagons for granted, holds the temples up as an example of the perfection of the "golden section".

4. The optical view according to which the Greeks planned the ratios applying the rules of perspective.

Concerning the facts there are arguments even inside these particular groups — e.g. in choosing the module (whether to take the lower column diameter or the intercolumniation); there are debates among those who are in favour of the geometrical interpretation (whether to choose the pentagon or the hexagon as the starting point), etc. The arguments naturally contain references to facts and figures but these are constantly disturbed either by the vagueness of data and measuring or by a wild neglect of the facts: by "bold" generalization of an example taken at random or (especially in the essays of the adherents of the "golden section") simple falsifications.

After all, every preconception can be proved if irregularities are explained by errors in building techniques or by larger or smaller distortion of facts (e.g. supposing the Attic foot to be 297 or 298 mm long instead of 296 mm). If, for example, our starting point is that the scales — due to the geometrically based construction — can be denoted in non-terminating decimal numbers then there is not a single ratio, and there cannot be one either, that could not be demonstrated with the help of some mathematical speculation. And if we consider trigonometrical ratios as further possibilities of forming scales, then we are giving free scope for subsequent wrong interpretations.

We cannot find a convincing system of theories — that is, one which respects the approximately exact data and is based on the total of the material free of mystical preconceptions and false modernity as well.

We have restricted our study to the Doric temples only: in the archaic and classic periods they formed the majority of buildings (we have hardly any reliable data of the Ionic temples at our disposal) and because of their "strictness" they are the most suitable bases of any generalization.

First of all, let us lay down the archeological data we regard as the basis of our critical examinations and positive proof.

Preliminary remarks: (cf. p. 283)

a) We have already noted that temples Nos 20. and 33. are excluded from our calculations. When analysing the material in the order dictated by history and history of culture, we shall disregard temple No. 5., as it cannot be entered into any of the big units because of its geographical isolation.

b) As can be seen from the catalogue above, the data are often uncertain (not to speak about the great differences in archeological literature concerning measurements) even in the case of such artistically and historically valuable masterpieces as e.g. the temple of Apollo at Delphi. The truth of every speculation taking absolutely exact measurements as its starting point, is highly questionable from the outset.

c) The period in which Doric temples were built lasted for over 700 years, and there are quite a number of them which were rebuilt after a few decades or centuries (e.g. the temple of Zeus Olympius at Athens). Not only building techniques but ideologies (and thus the functions of temples), tastes and styles also changed. All these are ignored in mathematical generalizations.

d) In our study we could have reduced the number of the buildings, already reduced to 47 from the original 49 ones, as No. 14. belongs to those the ground plan of which is of the 'in antis' type, Nos 45. and 48. of prostyle and Nos 36. and 49. of amphiprostyle buildings, so they are exceptions to the rule, where planning is based on the principles of the peripteral type. They hardly upset the general characteristic of the ratios of main dimensions, but in certain conspicuous cases we shall point out their deviations.

e) At least three main dimensions are needed to form a system of ratios (that is why similarly to No. 20., Nos 18. and 22. could have been omitted, too) and for a more complete generalization all the five ratios are demanded. When we shall analyse the ratio of width to other dimensions, only the flank : width table will contain all the data taken from the 47 temples. When stating the scales on the front, we had to make do with fewer data.

f) Buildings Nos 14., 33. and 45. are not temples but because of the similarity of principles in their planning they may be interesting in the examination of ratios as well.

The outline of working process

We wish to prove that the ratios of the main dimensions in Doric temples can be denoted in ratios of whole numbers and that this way of proportioning should be considered the basic planning principle. This is suggested, on the one hand, by the results of statistical generalization, and on the other hand, by the simplicity of the process which makes it acceptable from the point of view of history, too.

Our first task, however, is the critical analysis of traditional and current views found in archeological literature (i.e. the basic thesis of Vitruvius; the module-theory based on it; then the hypothesis that "module = practical unit of measurement" and finally the view referring to the ratios of poly-

The dimensions of Doric temples in meter units[1]

No.	Name of the building	1. Flank	2. Width	3. Height of columns	4. Height of order	5. Height of ridge (circ.)
1.	Olympia, Heraeum	50.010	18.750	5.220	—	—
2.	Syracuse, Apollo	55.330	21.570	7.980	—	—
3.	Syracuse, Olympieum	circ. 62.050	22.400	circ. 8.000	—	—
4.	Selinus 'C'	63.720	23.937	8.653	13.133	17.388
5.	Assos, Athena	30.310	14.030	4.780	6.800	9.170
6.	Corinth, Apollo	circ. 53.824	circ. 21.484	7.240	—	—
7.	Selinus 'D'	55.679	23.626	8.310	12.264	—
8.	Paestum, Basilica	54.270	24.510	6.445	—	—
9.	Selinus 'F' or 'FS'	cric. 61.880	24.370	circ. 9.110	circ. 13.065	17.575
10.	Athens, Athena (Peisistratid)	43.150	21.300	circ. 7.400	circ. 11.399	—
11.	Selinus, Apollo ('G' or 'GT')	circ. 110.120	50.070	circ. 14.690	circ. 21.250	—
12.	Acragas, Zeus Olympius	circ. 110.095	circ. 52.740	circ. 17.625	circ. 24.820	33.355
13.	Paestum, Demeter	32.880	14.541	6.127	circ. 8.780	10.190
14.	Delphi, Athenian Treasury	9.687	6.621	4.128	5.745	7.003
15.	Metapontum, Tavole Paladine	33.460	16.060	5.135	—	—
16.	Acragas, Heracles ('A')	67.040	25.284	10.070	13.780	?
17.	Delphi, Athena Pronaea	27.464	13.250	4.600	—	—
18.	Sunion, Poseidon (Old)	circ. 30.200	circ. 13.060	—	—	—
19.	Aegina, Aphaea	28.815	13.770	5.272	7.238	9.080
20.	Athens, Older Parthenon**	circ. 66.940	23.533	—	—	—
21.	Syracusae, Athena	circ. 55.020	circ. 22.000	8.710	circ. 12.610	—
22.	Himera, Nike	55.955	22.455	—	—	—
23.	Selinus, Hera ('E' or 'ER')	circ. 67.735	25.324	circ. -10.150	circ. 14.620	—
24.	Olympia, Zeus	64.120	27.680	10.430*	14.510	18.700
25.	Paestum, Poseidon	59.975	24.264	8.880	12.668	15.190
26.	Acragas, Hera Lacinia ('D')	38.100	16.910	6.360	circ. 9.260	?
27.	Selinus 'A'	40.303	16.129	6.235	9.015	?
28.	Bassae, Apollo	38.244	14.478	5.957	8.737	—
29.	Athens, Hephaesteum	31.769	13.708	5.713	7.733	9.433
30.	Athens, Parthenon	69.503	30.880	10.433	13.728	17.960
31.	Sunion, Poseidon (New)	circ. 31.124	13.470	6.024	8.034	?
32.	Athens, Ares	circ. 33.174	circ. 14.344	circ. 6.725*	circ. 8.752	—
33.	Athens, Propylaea***	—	—	—	—	—
34.	Rhamnus, Nemesis	21.420	circ. 9.966	circ. 4.100*	circ. 5.494	?
35.	Acragas, Concord	39.420	16.925	6.700	circ. 9.660	11.900
36.	Delos, Apollo (Athenian)	circ. 17.014	circ. 9.686	circ. 4.650	circ. 6.126	7.633
37.	Segesta	58.035	23.120	9.366	12.951	15.897
38.	Argos, Hera	circ. 36.900	circ. 17.305	circ. 7.400	circ. 9.880	—
39.	Epidaurus, Asclepius	circ. 23.060	circ. 11.160	circ. 5.200	circ. 6.720	8.820
40.	Delphi, Apollo	circ. 58.180	circ. 21.680	circ. 10.590	—	—
41.	Tegea, Athena Alea	47.550	19.190	9.474	11.895	13.980
42.	Nemea, Zeus	42.555	20.090	10.368	12.935	?
43.	Stratos, Zeus	32.420	16.570	circ. 7.095	circ. 9.166	?
44.	Olympia, Metroum	20.670	10.062	—	—	—
45.	Athens, Nicias Monument	circ. 15.220	11.095	circ. 5.102	circ. 6.569	?
46.	Delos, Apollo (peripteral)	circ. 28.530	12.470	circ. 5.200	circ. 7.260	?
47.	Pergamum, Athena Polias	21.770	12.270	5.260	6.485	8.470
48.	Pergamum, Dionysus	circ. 10.135	circ. 6.765	4.490	5.340	?
49.	Eleusis, Artemis Propylaea	circ. 12.330	circ. 6.440	4.530	5.949	?

* dimension measured on the front
** omitted because of uncertainty and scarcity of data
*** omitted from our calculations because of its complicated planning

gons). Our method will be partly the confrontation of statements with facts and partly carrying out examinations with the help of a computer.

I. Critical chapters

1. Checking the basic thesis of Vitruvius

Vitruvius generally considers the lower diameter of the columns, or, in the case of Doric temples, the half of it, such a common divisor *(modulus)* the multiplication of which gives as a result the measurements of main dimensions and the division of which defines those of smaller parts Regardless of the rational values of the module-theory, we are examining the ratios of the principal dimensions to these diameters (D) on Doric temples: whether they are exactly or approximately whole numbers and if we find values which are 0.5 greater or smaller than that whole number — in principle that is the same — then we shall decide whether the theory that "module = lower diameter" can be true.

The temples are marked here and later, too, with their code numbers to be found in the table. They are named only if it seems relevant from the point of view of proportioning.

(The figures given in italics may be considered the multiples — either exactly or approximately — of the lower diameter or radius.)

If we analyze statistics with mathematical strictness — that is, we demand whole numbers as quotients and accept deviations of a maximum of a few hundredths — it is possible to state the following:

The height of ridge of the temple at Assos, the width of the Basilica at Paestum, the flank and width of the temple of Nemesis at Rhamnus, the width of the temple of Asclepius at Epidaurus and of the temple of Apollo at Delphi and the height of order of the temple of Zeus at Stratos can be regarded as the multiples of the columnal diameter (D); the flank of the temple of Zeus at Olympia, the width of the temple of Poseidon at Paestum, the height of order of the temple of Apollo at Delos and the height of ridge of the temples at Epidaurus and Tegea can be regarded as the multiples of the lower radius of the columns.

The values we have as a result of these divisions are far from being adequate to prove the general (generalizable) truth of the module-theory of Vitruvius; in fact, there is not one single temple for the planning of which the theory is valid.

No.	D (mm)	Flank : D	Width : D	Height of columns : D	Height of order : D	Height of ridge : D
1.	1200 1280	41.675 39.070	15.625 14.648	4.350 4.078	— —	— —
2.	2010	27.527	10.731	3.970	—	—
3.	1840	33.723	12.174	4.348	—	—
4.	1910	33.361	12.532	4.530	6.876	9.104
5.	915	33.125	15.333	5.224	7.432	*10.022*
6.	1744	30.862	12.319	4.151	—	—
7.	1701	32.733	13.889	4.885	7.210	—
8.	1442	37.635	*16.997*	4.469	—	—
9.	1790	34.570	13.615	5.089	7.299	9.818
10.	1630	26.472	13.067	4.540	6.993	—
11.	2970	37.077	16.859	4.946	7.155	—
12.	4050	27.184	13.022	4.263	6.128	8.236
13.	1267	25.951	11.477	4.836	6.930	8.043
14.	759	12.763	8.723	5.439	7.569	9.227
15.	1060	31.566	15.151	4.844	—	—
16.	2085	32.153	12.127	4.830	6.609	—
17	1005	27.327	13.184	4.577	—	—
18.	980	30.816	13.327	—	—	—
19.	989	29.135	13.923	5.331	7.319	9.181
20.	—	—	—	—	—	—
21.	1920	28.656	11.458	4.536	6.446	—
22.	1875	29.843	11.976	—	—	—
23.	2268	29.866	11.166	4.475	6.446	—
24.	2250	*28.498*	12.302	4.636	6.449	8.311
25.	2112	28.397	*11.489*	4.205	*5.998*	7.192
26.	1387	27.469	12.192	4.585	6.676	—
27.	1320	30.533	12.219	4.723	6.830	—
28.	1161	32.941	12.470	5.131	7.525	—
29.	1018	31.207	13.466	5.612	7.596	9.266
30.	1905	*36.485*	16.210	5.477	7.206	9.428
31.	1043	29.841	12.915	5.776	7.703	—
32.	1100	30.158	13.040	6.111	7.956	—
33.	—	—	—	—	—	—
34.	714	*30.000*	*14.000*	5.742	7.695	—
35.	1452	27.149	11.656	4.614	6.653	8.196
36.	814	20.902	11.899	5.713	7.526	9.377
37.	1955	29.685	11.826	4.791	6.625	8.131
38.	1320	27.955	13.110	5.606	7.485	—
39.	930	24.796	*12.000*	5.591	7.226	*9.484*
40.	1806	32.215	*12.004*	5.864	—	—
41.	1550	30.674	12.381	6.112	7.674	*9.019*
42.	1630	26.107	12.325	6.358	7.936	—
43.	1310	24.748	12.649	5.416	*6.997*	—
44.	850	24.318	11.838	—	—	—
45.	844	18.033	13.146	6.045	7.783	—
46.	945	30.190	13.196	*5.503*	7.683	—
47.	754	28.873	16.273	6.976	8.601	11.233
48.	620	16.347	10.911	7.242	8.613	—
49.	782	15.767	8.235	5.793	7.607	—

If, however, we evaluate the results loosely then, of course, the number of "correct results" will be multiplied. On this basis it is customary, for example, to take the ratio of the columns on the Parthenon to be 5.5, rounding off the value 5.477 (height : diameter).

Surely, every bigger component of a building will be inevitably different in practice from the plan made on the drawing desk, and so their ratios will necessarily differ, too.

There is not a single Greek temple at least the main dimensions of which are proportionable with mathematical exactness — e.g. width to architrave and lower tympanum line, intercolumniation on the front to that on the flank or length of the tympanum to its height. That is one of the reasons why we have restricted our study to the main dimensions which are also statically of fundamental importance.

It is certainly debatable, however, how far the loosening of exact ratios may go, whether it is justifiable to round off e.g. the ratio of the height of order of the Parthenon to the lower diameter of the columns to 7, instead of 7,206. In our opinion, values cannot be dealt with so liberally, *firstly* because in this way everything could be explained, *secondly* because statistics do not show any tendency of adjustment, *thirdly* because taking the diameter as the starting point (e.g. in defining the flank of the Parthenon) would have neither statical nor aesthetical reasons, *fourthly* because we cannot find a system based on this unit anywhere (e.g. it is impossible to account for the width of the Parthenon on such a principle).

Even if some of the ratios pointed out in certain temples can be described as the multiples of the lower diameter or radius (ignoring even the contraction of the external columns seen on many buildings and the frequent differences in measurement between the columns on the front and on the flank), our data — or in other words, the facts — definitely contradict the module-theory suggested by Vitruvius.[2] *The diameter of columns did not serve as a common unit of measurement in planning the temples, in relating the main dimensions.*

The cases when the quotients (ratios) are exactly whole numbers, could not be accidents: the architects must have planned some of the dimensions of those temples deliberately but these sporadic data do not permit us to generalize this method either according to periods, or to the geographical situation of the temples.

2. *Looking for module with a computer*

Our aim is to find a common unit of scale in the main measurements, and if the facts do not prove its existence, then to furnish a basis for further examinations.

Our hypothesis is the following: If there is — even in the case of only one temple — a unit (module) suitable for generating all the dimensions, then it must be among the common divisors of the main measurements chosen by us.

We included in our examinations the following main dimensions considered characteristic:

1. Length of flank: a dimension forming a right angle to the front of the temple, at the height of the pedestal, 2. Width: the length of the front at the height of the pedestal, 3. Height of columns: the distance between the stylobate and the lower level of the architrave, 4. Height of order: the distance between the stylobate and the upper level of the horizontal cornice, 5. Height of ridge: the distance between the stylobate and the ridge of the tympanum.

Knowing the practice of building techniques, we had to suppose that the gauged measurements, accurate to the millimetre, do not exactly correspond with the data originally planned. Size tolerance has been defined in accordance with the above order of dimensions as follows: 1. ± 200 mm, 2. ± 100 mm, 3. ± 50 mm, 4. ± 100 mm, 5. ± 150 mm. With the help of the computer we tried to find common divisors among the values corrected with these tolerances.[3]

The logical construction of the program:

We supposed that in train of planning the main measurements were established first. As starting point we chose the width of the temples from among the possible dimensions: we were trying to find out which divisors of the width go into the other measurements examined without a remainder. In theory there is an infinite number of such "particles"; for instance, if we suppose that 1 mm is the common divisor (module), it goes into every dimension without a remainder — in case we give the lengths in millimetres, not in smaller units. This, however, is impossible as the fundamental unit of planning had to be near one of the units of length used in Greek antiquity. We know that the Attic foot was somewhere between 294 and 296 mm,[4] so rounding it off, we defined the low limit for the *least* common divisor as 290 mm. If we subtract the maximum size tolerance given from the length measured, and divide it by 290 mm, we receive the value that will show us how many times the smallest chosen unit is less than the particular dimension of the building being studied. For instance, the width of the Parthenon is 30,880 mm; subtracting the maximum size tolerance, 30,880—100 = 30,780 mm is what remains. 30,780:290 = 106.1379; that is to say, the length in question is approximately 106 times as long as our 290 mm.

The *greatest* possible unit was chosen as the maximum size tolerance added to the unit we had for a starting point and divided by two. In our example this value is obtained as follows: 30,880 + + 100 = 30,980 mm; 30,980:2 = 15,490 mm, since it is obvious that the measurement "width + size tolerance" cannot be the exact multiple of any unit greater than this value.

Following this, we asked the computer:

Is there any measurement on all the five main dimensions — corrected with the size tolerances — the value of which is an exact multiple of any number between 290 and the half of the greatest measurement calculated?

In the program, using the data of our previous example, we solved the problem as follows:

The range examined in the case of the Parthenon is between 30,780 and 30,980 mm. The lower limit of the corrected dimension contains the smallest unit 106 times, the respective figure for the upper limit and the largest unit is 2. As a first step, the computer had to find a number which gives a whole number if divided by two. If there is a number like this — in our example 30,780:2 = = 15,390 —, then the program should find out whether the range of the next given dimension (flank of the temple) — corrected with size tolerance — contains any multiples of 15,390. (In the range between 69,303 and 69,703 any number divided by 15,390 = whole number?)

If a number like this does exist, then the computer examines the third given dimension, the height of columns. Our question — similarly to the previous one — was the following: Is there a number within the given limits which is an exact multiple of the result we had when examining the width (in our example 15,390)? If such a number does not exist, then the computer goes back to the examination of the width. This is repeated until (1) there is a whole number again as the result of the process or until (2) it reaches the given high limit.

In case (1) it examines the height of columns again as described above. If it finds no quotient which is a whole number, it means this is case (2), then the examination of the width is resumed where it stopped before the examination of the flank.

What happens if we find more than one quotient which are whole numbers in the course of the examination of both the flank and the height of columns? Well, in this case the computer continues the program and finds out whether the divisor we had so far gives an exact quotient, a whole number, in the examination of the height of order and height of ridge, too. As an example we shall introduce the results of this process in the case of four buildings.

I. In the examination of the temple of Zeus at Olympia (No. 24.) we found 40 common divisors among the dimensions of width, flank and height of columns and 79 among the dimensions: width, flank and height of order. The common divisors of all the main dimensions are: 521, 455, 454, 374, 373, 347, 338, 315, 308, 307, 298, 297, 291 mm — altogether 13 values.

II. Among the dimensions of width, flank and height of order in the case of the temple of Poseidon at Paestum (No. 25.) there are 54 common divisors, and among the dimensions: width, flank and height of order there are altogether 98. The common divisors of all the five main dimensions are: 1,275, 1,274, 1,273, 1,272, 637, 636, 425, 424, 372, 318, 316, 307, 296, 295 — altogether 14 values.

III. When examining the Parthenon (No. 30.), among the dimensions: width, flank and height of columns we found 46, and among the dimensions: width, flank and height of order 100 common divisors. We extended then the examination to the fifth main dimension, to the height of ridge, using the same method. In order to illustrate our method, we publish the complete material. The common divisors which are valid for all the five main dimensions (within the given size tolerances) are in italics. The marks *d* (deviation) indicate ± deviations from the lengths actually measured.

In this way we found altogether 20 common divisors. As, however, two values (335 and 299) occur twice, their number is actually 18. (Needless to say, we selected the data in a similar manner when examining all the other temples, too.) To receive the results above the computer would have had to carry out 1 million 125 thousand operations. This amount of problems could be reduced to 11,080 operations with the help of considerations expounded in the introduction to the method.

IV. When studying the temple of Concord (F) at Acragas (No. 35.) among the dimensions: width, flank and height of columns we found 47 common divisors, and 97 among the dimensions: width,

	Width	Flank	Height of columns	Height of order	Height of ridge	Common divisor
	30,758 mm	69,430 mm	10,480 mm	13,755 mm		655 mm
d	— 95 mm	— 73 mm	+ 47 mm	+ 27 mm		
	30,856	69,426	10,469	13,775		551
d	— 24	— 77	+ 36	+ 47		
	30,810	69,678	10,428	13,746	18,012 mm	*474*
d	— 70	+ 175	— 5	+ 18	+ 52 mm	
	30,875	69,350	10,450	13,775	18,050	*475*
d	— 5	— 153	+ 17	+ 47	+ 90	
	30,940	69,496	10,472	13,804	18,088	*476*
d	+ 60	— 7	+ 39	+ 76	+ 128	
	30,940	69,615	10,465	13,650		455
d	+ 60	+ 112	+ 32	— 78		
	30,784	69,472	10,400	13,728	17,888	*416*
d	— 96	— 31	— 33	0	— 72	
	30,858	69,639	10,425	13,761	17,931	*417*
d	— 22	+ 136	— 8	+ 33	— 29	
	30,932	69,388	10,450	13,794	17,974	*418*
d	+ 52	— 115	+ 17	+ 66	+ 14	
	30,877	69,373	10,426	13,634	18,045	*401*
d	— 3	— 130	— 7	— 94	+ 85	
	30,954	69,546	10,452	13,668	18,090	*402*
d	+ 74	+ 43	+ 19	— 60	+ 130	
	30,793	69,377	10,388	13,727		371
d	— 87	— 126	— 45	— 1		
	30,876	69,546	10,416	13,764	17,856	*372*
d	— 4	+ 43	— 17	+ 36	— 104	
	30,959	69,378	10,444	13,801	17,904	*373*
d	+ 79	— 125	+ 11	+ 73	— 54	
	30,874	69,646	10,411	13,642	17,950	*359*
d	— 6	+ 143	— 22	— 86	— 10	
	30,960	69,480	10,440	13,680	18,000	*360*
d	+ 80	— 23	+ 7	— 48	+ 40	
	30,820	69,345	10,385	13,735	18,090	*335*
d	— 60	— 158	— 48	+ 7	+ 130	
	30,820	69,680	10,385	13,735		335
d	— 60	+ 177	— 48	+ 7		
	30,912	69,552	10,416	13,776		336
d	+ 32	+ 49	— 17	+ 48		
	30,875	69,550	10,400	13,650	17,875	*325*
d	— 5	+ 47	— 33	— 78	— 85	
	30,970	69,438	10,432	13,692	17,930	*326*
d	+ 90	— 65	— 1	— 36	— 30	
	30,906	69,462	10,404	13,770		306
d	+ 26	— 41	— 29	+ 42		
	30,797	69,368	10,465	13,754	17,940	*299*
d	— 83	— 135	+ 32	+ 26	— 20	
	30,797	69,667	10,465	13,754		299
d	— 83	+ 164	+ 32	+ 26		
	30,888	69,498	10,395	13,662	17,820	*297*
d	+ 8	— 5	— 38	— 66	— 140	
	30,846	69,549	10,476	13,677	18,042	*291*
d	— 34	+ 46	+ 43	— 51	+ 82	

flank and height of ridge. The common divisors of all the five main dimensions are the following: 514, 446, 445, 441, 395, 394, 393, 392, 354, 353, 352, 351, 337, 321, 320, 319, 318, 306, 292, 291 mm — altogether 20 values.

It seems unnecessary to publish the complete mathematical documentation of all the other temples as well because our calculations can be easily checked. We indicate only the number of common divisors of each temple and mark the least and greatest common divisors in millimetres. (For the complete list of common divisors see the part II. of the Appendix.) — The order of the table is: Code number, number of common divisors (in the 1. column), the least and greatest divisor in mm (in 2. and 3. columns).

a) *If all the five main dimensions were measured:*

No.	1.	2.	3.	No.	1.	2.	3.
4.	14	298	725	29.	11	299	406
5.	17	297	482	30.	18	291	655
9.	15	304	763	35.	20	291	514
12.	16	293	556	36.	19	291	515
13.	17	304	441	37.	15	293	723
14.	16	297	594	39.	12	303	741
19.	21	291	659	41.	8	295	410
24.	13	291	521	47.	8	291	431
25.	14	295	1,275				

b) *In case of four main dimensions:*

7.	18	298	641	31.	12	299	406
10.	16	308	817	32.	10	297	420
11.	19	294	735	34.	18	292	458
16.	16	295	722	38.	55	297	2473
21.	14	300	788	42.	18	295	547
23.	16	291	634	43.	18	295	505
26.	19	290	708	45.	18	298	734
27.	16	298	897	46.	28	303	652
28.	21	296	662	48.	13	298	516
				49.	11	303	521

From our table it is evident that there are several common divisors among the main dimensions of each temple, and this fact refutes the module theory completely, as the existence of numerous common divisors precludes the possibility of treating any of them as "*the* module". If the main dimensions of a building have many different divisors (that is, there are numerous smaller measurements of which — according to the notion of module — the main dimensions are exact multiples), then any of them may be accepted as "potential modules".

The temple of Hera at Argos (No. 38.) is the most conspicuous example to show that it is impossible to give preference to any of the 55 common divisors. But the temple at Tegea and the temple of Athena at Pergamum (Nos 41. and 47.) which have the fewest common divisors, 8 in number, prove the same.

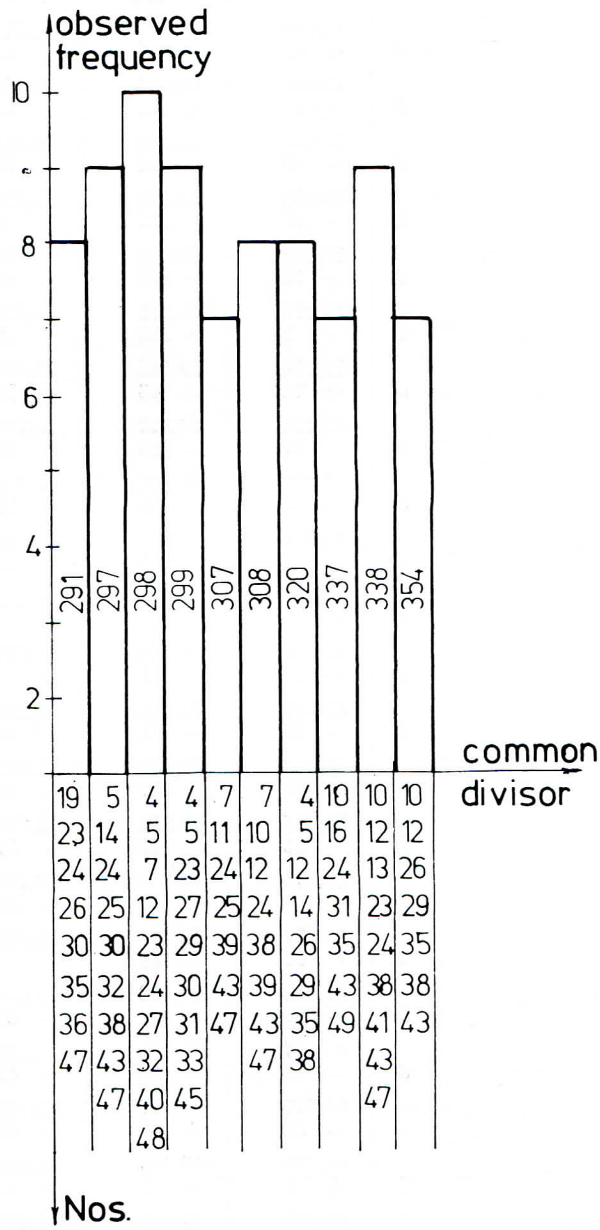


Fig. 1

The equal importance in terms of mathematics of common divisors found among the main dimensions of each temple means that we have no proof and right to choose one or the other as the module. We may not state e.g. that the measurement 297 mm was the fundamental unit in planning the Parthenon because the same could be said of 17 other — mathematically equally correct — meaurements, beginning with 291 mm and up to 655 mm.

We try to make our choice more realistic by examining which are the common divisors occurring most frequently — whether one or another of them prove to be characteristic of temples built in a certain period or territory.[5] The ten most frequent common divisors can be seen in *Figure 1.* where the Nos identify the temples in question. Preparing a complete histogram would be unnecessary because the common divisors occurring only once or in very few cases (and naturally these are found most often among the greater values) give no basis for generalization.

In order to be able to consider these common divisors as generally and consciously used modules, and to be able to choose one or the other without any hesitation, it is necessary that the temples themselves — the main dimensions of which are multiples of the module — should be historically related to each other. Let us study the range between 297 and 300 mm from this latter point of view:

We have found some surprising coincidences. We may establish as a fact that (1) in case of the temples at Selinus the common divisor is 298 mm or 1 or 2 millimetres greater, (2) that in the case of the main dimensions of the temples built in or

297 mm:	Assos, Athena	about	540 B.C.	Asia Minor
	Delphi, Treasury		507	Balkan Peninsula
	Olympia, Zeus		468—460	Peloponnesos
	Paestum, Poseidon	about	460	South Italy
	Athens, Parthenon		447—432	Athens
	Athens, Ares		440—436	Athens
	Argos, Hera		432—416	Peloponnesos
	Stratos, Zeus	about	321	Balkan Peninsula
298 mm:	Selinus "C"	about	550—530	South Italy
	Assos, Athena	about	540	Asia Minor
	Selinus "D"	about	535	South Italy
	Acragas, Zeus	about	510—409	South Italy
	Selinus "E"	about	480—460	South Italy
	Olympia, Zeus		468—460	Peloponnesos
	Selinus "A"	about	460	South Italy
	Athens, Ares		440—436	Athens
	Athens, Nicias Monument		319	Athens
299 mm:	Selinus "C"	about	550—530	South Italy
	Assos, Athena	about	540	Asia Minor
	Selinus "E"	about	480—460	South Italy
	Selinus "A"	about	460	South Italy
	Athens, Hephaesteum		449—444	Athens
	Athens, Parthenon		447—432	Athens
	Sunion, Poseidon		444—440	Near Athens
	Athens, Ares		440—436	Athens
	Athens, Nicias Monument		319	Athens
300 mm:	Selinus "C"	about	550—530	South Italy
	Selinus "G"	about	520—450	South Italy
	Syracusae, Athena		480	South Italy
	Athens, Hephaesteum		449—444	Athens
	Sunion, Poseidon		444—440	Near Athens
	Athens, Ares		440—436	Athens

near Athens, in the middle of the 5th century B. C., the common divisor is also 297 mm or 1 or 2 millimetres greater. Other occurrences of these values may be accidental (in distant regions or centuries earlier or later), again others may reinforce the statement (the vicinity of Paestum, Acragas and Syracusae to Selinus). The validity of the statement is questioned, however, by the conspicuous irregularities of the temples at Selinus, built in the 6th century B. C. and also by the defects resulting from primitive building techniques (e.g. those in intercolumniations).

The common divisors of the temples at Selinus built in the 5th century, make the module-hypothesis probable. The same cannot be said so definitely of the temples at Athens built in the classical age. In order to be able to consider the length 299 mm as the common module — or generally applied unit of measurement around 440-430 B. C. — for the Hephaesteum, the Parthenon, the temple of Ares and the temple of Poseidon at Sunion — it is also necessary that no other common divisor should be found when dividing the main dimensions of all the buildings. But this is not the case, as the dimensions of three of the four temples (Hephaesteum, Parthenon and the temple of Poseidon at Sunion) are the exact multiples of 335 mm as well.

The fact that the circle in which there is a possibility of finding a module is reduced to such a little extent means that it is impossible to declare that one certain module was applied, even if within particular cultural complexes.

Let us check now what the function of supposed (or only probable) modules could possibly be in planning the buildings. Being more practical: let us see how many times as long as 298 and 299 mm the main dimensions of the temples at Selinus ("E" and "A") are, and how many times as long as 299 and 335 mm (common divisors) the main dimensions of the Hephaesteum, Parthenon and the temple of Sunion are.

The results we have are surprising, nay, unbelievable. (In brackets we have given the deviations from the data we used; they are due to the fact that common divisors were multiplied by whole numbers.) It is highly improbable both technically and from the point of view of psychology of labour that temples were designed and built in such an extremely complicated way. For applying this method, first of all, the Greek masters would have needed such "millimetre paper" where

		Selinus "E"	Selinus "A"
		the common divisor goes into the dimensions	
298 mm	flank	227 (−89)	135 (−73)
	width	85 (+ 6)	54 (−37)
	height of columns	34 (− 18)	21 (+23)
	height of order	49 (− 18)	30 (−75)
	height of ridge	58 (−104)	
299 mm	flank	227 (+138)	135 (+62)
	width	85 (+ 91)	54 (+17)
	height of columns	34 (+16)	21 (+44)
	height of order	49 (+31)	30 (−45)
	height of ridge	58 (−46)	

	Hephaesteum	Parthenon	Sunion, Poseidon
	the common divisor goes into the dimensions		
299 mm flank	106 (− 75)	232 (−135)	104 (−28)
width	46 (+ 46)	103 (− 83)	45 (−15)
height of c.	19 (− 32)	35 (+ 32)	20 (−44)
height of o.	26 (+ 41)	46 (+ 26)	27 (+39)
height of r.	31 (−164)	60 (− 20)	
335 mm flank	95 (+56)	207 (−158)	93 (+31)
width	41 (+27)	92 (− 60)	40 (−70)
height of c.	17 (−18)	31 (− 48)	18 (+ 6)
height of o.	23 (−28)	41 (+ 7)	24 (+ 6)
height of r.	28 (−53)	53 (−205)	

the ground plan of the Parthenon would have been drawn on 232×103 or 207×92 small squares, and the sketch of its front (representing the width, height of columns, height of order and height of ridge) would have been made on 103×60 or 92×53 small squares if the builders drew the temple to scale on the basis of the supposed module.[6] Yet, such high-level precision was not at all possible by contemporary writing materials. The perfectly precise enlargement of the sketch in practice would have been equally difficult, as for instance even the cell of the Parthenon is not an exact rectangle as it should be, and it is noticeable how inexact all the axial spacings are.

In an aesthetical sense it is meaningless (and would have been meaningless then too) if — keeping the same example — the height of columns, height of order, height of ridge, width and flank had been planned in the ratio of 35:46:60:103:232.

However convincing the numbers may be, we must admit that if we consider this mathematical possibility, it seems to be improbable, both technically and aesthetically.

The use of the values we got as the result of mathematical operations, is improbable too, because none of them coincides with any of the contemporary units of measurement. The possibility namely, that architects used a specific, mystical unit of measurement, as for example the length 335 mm, should be excluded.

And if we finally examine those temples where we had other — mathematically equally correct — common divisors (e.g. 338 and 354 mm), we find that they cannot be related historically either, and the common divisors are even more different from the units of measurement actually used in antiquity.

It is natural that the more we reduce size tolerance the fewer the number of common divisors will be.[7]

The mere fact that in most cases the lower diameter of columns and axial spacing on the front are different from those on the flank, and also, the fact that these values may differ even on one side because of the thicker or contracted external columns, contradicts the module-theory interpreted as rigorously as Vitruvius did.

Thus we went on with our research only to be absolutely objective: we wished to find out whether it was not possible that some of the common divisors found with the help of the computer were exact divisors of secondary dimensions, and as such might be "promoted" to the rank of "module".

We extended our investigation in two directions. First (A.) we examined which common divisors of the main dimensions are approximately exact divisors of the axial spacings and column diameters of four representative buildings: temple of Zeus at Olympia (No. 24.), the temple of Poseidon at Paestum (No. 25.), the Parthenon (No. 30.), the temple of Concord at Acragas (No. 35.). Secondly (B.) we examined whether the common divisors found in the case of the Athenian-Attic temples of the 5th century (297, 298, 299 and 300 mm) are exact divisors of axial spacings and column diameters of the same temples.

A

No. 24.: 521 mm goes into the secondary dimensions twice (into the axial spacing of middle columns on the front and on the flank), so does 374 mm (into the axial spacing and diameter of middle columns on the front), and also 373 mm (into the same measurements as 521 mm); only once do 454 and 307 mm go into them.

No. 25.: 1,275, 1,274, 637, 636 and 372 mm go into the measurements in question once.

No. 30.: 306 mm goes three times (into the axial spacing of middle columns on the front and on the flank and into the axial spacing of the external columns on the front-side); 475, 474, 326 and 325 mm go into the measurements only once.

No. 35.: 291 mm goes three times (into the axial spacing of middle columns on the front and on the flank and into the diameter of the same columns on the front-side); 445, 320, 319, 292 mm go into the measurements once.

The secondary dimensions taken into consideration (depending on the differences of size between columns on the front viz. on the flank and differences between middle and external columns) functioned as dividends at least 6, sometimes 7 or even 8 times in the ratios. As divisors we applied all the measurements we found as common divisors of the main dimensions of the temples examined with the computer. Bearing this fact in mind, three occurrences of one measurement (as 306 mm in the case of the Parthenon and 291 mm in the case of the temple of Concord at Acragas) are nothing more than mathematical coincidences.

It would be a bold conclusion if on this basis (that is, making use of the "selective function" of secondary dimensions) we were to choose the measurements 306 mm and 291 mm as the "true modules" of the Parthenon (that has altogether 18 common divisors) and the temple of Concord at Acragas (that has 20 common divisors). We should immediately add that examining the temple of Zeus at Olympia and the temple of Poseidon at Paestum with the same method, we came to a complete failure, and that the scattering of data could not be explained in any way.

B

Among the secondary dimensions of temples No. 29. and No. 30. there is not one which is the exact multiple of any value in the range between 297 and 300 mm.

In the case of temple No. 31., the axial spacing of external columns and the lower diameter of middle columns on the front are exact multiples of 297 and 298 mm, and the axial spacing of middle columns in temple No. 32. is that of 299 and 300 mm. These are obvious coincidences: this much is not enough to prove the similarity of planning principles in building the Athenian-Attic temples of the 5th century.

The results of our examinations proceeding in two directions, are all negative. In this way we have proved our former intuition, namely that we are unable to prove the module-theory even if — taking into consideration the common divisors of main dimensions — the analysis is extended to the secondary dimensions.

Summarizing the results of our examination done with the computer, the following can be said:

1. In accordance with the theory of probability, the buildings had a great number of common divisors in each case.

2. It is impossible to find a definite mathematical relation between the common divisors even within certain groups of temples. According to our supposition we expected to have only a few independent common divisors at the very most, and we hoped that all the others would be mathematically related to at least one of them, that is, it would strengthen the possibility that this particular one must have been the common unit of measurement valid for all the dimensions, so it must have been *the* module. But this supposition has proved wrong.

3. The common divisors of the main dimensions on the buildings examined bear no relation to each other. If we suppose, though, that there must be some connection between the temples built in the same region and period (as planning customs and traditions must have been the same), we should have had to find such groups of values that could be related within one or the other group of temples. Such relations are rare (on the temples of the 5th century Selinus and Attica) and they are not at all sure, only possible.

3. *Refutation of the hypothesis "module = practical unit of measurement"*

On the island of Samos the temple erected to worship Hera, and in Athens the old temple to worship the goddess of the city, were referred to as "100 feet long", and probably not without reason: the architect must have chosen the unit of length used in practice as his fundamental unit of planning. The name was used even later, in the classical age, but it did not exactly conform to the facts any more: the length 100 feet as an ideal dimension was associated with the beauty of a stately building — without any of its dimensions being this long. We do not know one single temple having been built to be 100 feet wide or long.

The idea itself that architects must have chosen one of the common units of length as the fundamental unit of defining the main dimensions, seems to be reasonable because enlarging to scale and building the temples must have been easier in this way. However attractive this thesis may be, the facts reflect something else: the number of temples where "module = unit of measurement" can be proved, is insignificant and even these buildings may have some other common "modules".

We cannot delimit the use of ancient Greek units of length with absolute certainty either. Only one thing is sure, namely that in classical Attica — but presumably everywhere else where the civilization of Athens was the ideal model to be followed — the most widely used among the smaller units of length was the approximately 296 mm Attic foot. It is obvious, however, that neither an outdated unit of measurement (e.g. the 294 mm foot), nor a unit of measurement unknown in their culture (e.g. the Ionic foot) could have been chosen as the basis for Doric planning. Let us take the well-known units of measurement one by one, and see which are identical with common divisors of the main dimensions of certain temples.

1. Doric foot	328	mm
2. Doric foot	327	mm
3. Old Attic foot	294	mm
4. Attic foot	296	mm
5. Old Ionic foot	349.5	mm
6. Ell	444	mm

328 mm: common divisor on temples No. 19. and No. 46., but they are not related in any historical or cultural way.

327 mm: common divisor on temples No. 11, No. 19. and No. 46. See remarks at the previous measurement.

294 mm: common divisor on temples No. 11. and No. 34. It could be the module of the former as well as the 327 mm Doric foot could be, so this is nothing more than a mathematical coincidence. In the latter case its being a consciously chosen unit, is out of the question.

296 mm: common divisor on temples No. 25., No. 28., No. 41., No. 42. and No. 43. Theoretically it is possible, but compared to the practical universality of the Attic foot, it is surprising how rarely it occurs in the classical age; it should be also noted that of No. 25, No. 41., No. 42. and No. 43. — 295 mm is also a common divisor, of No. 25. and No. 43. — 307 mm is another common divisor, of No. 41. and No. 42. — 315 mm is a common divisor, and finally of No. 25., No. 28. and No. 42. — 372 mm is an equally correct divisor. Let us suppose, however, that the measurement of the Attic foot was a few milimetres longer, that is to say, 297 mm is the hypothetical fundamental unit of planning: this is also possible, but (as we pointed out when examining the most frequent common divisors) its validity cannot be proved unquestionably even for the temples of classical Attica.

349 mm: common divisor on temples No. 21., No. 27., No. 32. and No. 46. but it cannot be proved either mathematically, or historically that it was used on principle; the same refers to the common divisor 350 mm, which occurs only on temples No. 21., No. 32. and No. 41.

444 mm: common divisor on temples No. 35. and No. 37. where the measurements 392 mm and 445 mm are also common divisors.

None of the units of length used in practice, is among the most frequent common divisors, and we were not able to prove that one or another of these was chosen by the architect to be the basic unit of planning a certain temple. Their occurrences are no more frequent, and from the historic and artistic points of view, they are not at all more probable than several other common divisors which are not related to any of the Greek units of length — that is, they must be regarded as coincidences.

4. *A false game with polygons*

The last point we shall examine critically is to refute the views on building construction based on geometric principles.

It is established that in certain cases scales can be expressed more precisely with values derived from geometric lengths than with ratios of whole numbers. These lengths can be determined numerically only by infinite decimal fractions (irrational numbers) but these were excluded from the notion of number by the Greeks — though they were able to construct such lengths easily, e.g. the value of

No. Ratio of measurements		Irrational quantity	
4. width : height of ridge	= 1.8219	$\frac{\sqrt{5}+\sqrt{2}}{2}$	= 1.8225
5. h. of ridge : h. of column	= 1.9184	$1:2+\sqrt{2}$	= 1.9142
9. flank : width	= 2.5392	$2+\sqrt{5}:2$	= 2.5322
9. width : height of order	= 1.8653	$1+\sqrt{5}:2$	= 1.8660
10. widht : height of order	= 1.8686	$1+\sqrt{2}:2$	= 1.8660
12. width : height of order	= 2.1249	$3\sqrt{2}:2$	= 2.1213
12. width : height of ridge	= 1.5812	$\frac{\sqrt{3}+\sqrt{2}}{2}$	= 1.5731
12. h. of ridge : h. of column	= 1.8925	$1:2+\sqrt{2}$	= 1.9142
14. h. of order : h. of column	= 1.4159	$\sqrt{2}$	= 1.4142
14. h. of ridge : h. of column	= 1.6957	$1+\sqrt{2}:2$	= 1.7071
16. h. of order : h. of column	= 1.3684	$\frac{1+\sqrt{3}}{2}$	= 1.3660
19. width : height of ridge	= 1.4051	$\sqrt{2}$	= 1.4142
19. h. of ridge : h. of column	= 1.8589	$1+\sqrt{3}:2$	= 1.8660
23. width : height of order	= 1.7321	$\sqrt{3}$	= 1.7321
25. width : height of order	= 1.9169	$1:2+\sqrt{2}$	= 1.9142
25. width : h. of column	= 2.7324	$1+\sqrt{3}$	= 2.7321
25. h. of ridge : h. of column	= 1.7106	$1+\sqrt{2}:2$	= 1.7071
25. h. of ridge : h. of order	= 1.1991	$\frac{1+\sqrt{2}}{2}$	= 1.2071
26. width : h. of order	= 1.8261	$\frac{\sqrt{5}+\sqrt{2}}{2}$	= 1.8225
31. width : h. of column	= 2.2361	$\sqrt{5}$	= 2.2361
32. width : h. of column	= 2.2859	$\sqrt{2}+\sqrt{3}:2$	= 2.2802
34. width : h. of column	= 2.4380	$\sqrt{3}+\sqrt{2}:2$	= 2.4392
35. widht : h. of ridge	= 1.4223	$\sqrt{2}$	= 1.4142
41. widht : h. of order	= 1.6133	$\frac{1+\sqrt{5}}{2}$	= 1.6180
41. widht : h. of ridge	= 1.3726	$\frac{1+\sqrt{3}}{2}$	= 1.3660
42. flank : width	= 2.1182	$1+\sqrt{5}:2$	= 2.1180
47. h. of ridge : h. of column	= 1.6103	$\frac{1+\sqrt{5}}{2}$	= 1.6180
49. flank : width	= 1.9146	$\frac{1+\sqrt{2}}{2}$	= 1.9142

$\sqrt{2}$ with a diagonal of the square, $\sqrt{3}$ with the height of a sectional side length of a regular triangle, $\sqrt{5}$ with a two section long median belonging to the perpendicular sides of an isoceles triangle, etc.

As an example let us take the temple of Hera at Selinus ("E", No. 23.), where width : height of order is 1.7321 in value, which is equal to 4 figures to the value of $\sqrt{3}$. We similarly get a more exact value than when approaching whith whole numbers, if we take the ratio width : height of column (2.2361) of the (new) temple of Poseidon at Sunion as $\sqrt{5}$: here too the value is exact to 4 figures. And all this is given without tolerance or rounding!

So the presumption is obvious that the construction of geometrical principles could be generalized — especially since the classical age when every mathematically trained engineer knew the ratios of regular polygons and forms.[8]

In the table on p. 293 (see also Appendix I.) we enumerate all those scales which can be exactly or approximately described with irrational numbers expressing geometric length.

Nothing is clear in the table above except an opaque mess. There is no mathematical quantity that could not be exactly or approximately expressed with some kind of clever speculation. Thus by irrational numbers the consciencious construction of each ratio can be proved — not only in architecture but also in sculpture or ceramics. But this is theoretically and practically (that is for technical reasons) impossible. Only then could we regard construction of temples on geometrical principles possible if some kind of common and not too complicated principles or tendencies showed in the formulas. But nothing like this can be found — on the contrary: the presumption is ridiculous rather then convincing, e.g. that the ratio width : height of order of the temple of Zeus at Acragas was constructed by the formula $\frac{3\sqrt{2}}{2}$ with perfect precision.

Only one or two scales of a few temples seem to have been constructed on a geometrical principle — the others mean accidental curiosities. Only in the case of temple No. 25. could we find four such scales that recollect this principle (here because of the simplicity of the construction this method was probably used) elsewhere the data are so scattered that no generalization or conclusion can be allowed.

The negative is also important in one respect: we have found only two such component-pairs among the 47 examined temples, altogether 339 ratios of measurements (the ratio of the width : : height of order on the Tegean Athena temple, No. 41., and height of ridge: height of columns on the temple of Athena at Pergamum, No. 47), which can be expressed by the golden section $\left(\frac{\sqrt{5}+1}{2} = 1.618\right)$.

Those researchers who consider that the geometric principle construction is due to the Greeks claim that the ground plan or front plan of temples (or at least of certain temples) correspond to the side-ratios of a regular pentagon or hexagon drawn in the circle.[9]

Let us take the possible variaties:

1. Ground plan (see Fig. 2.)

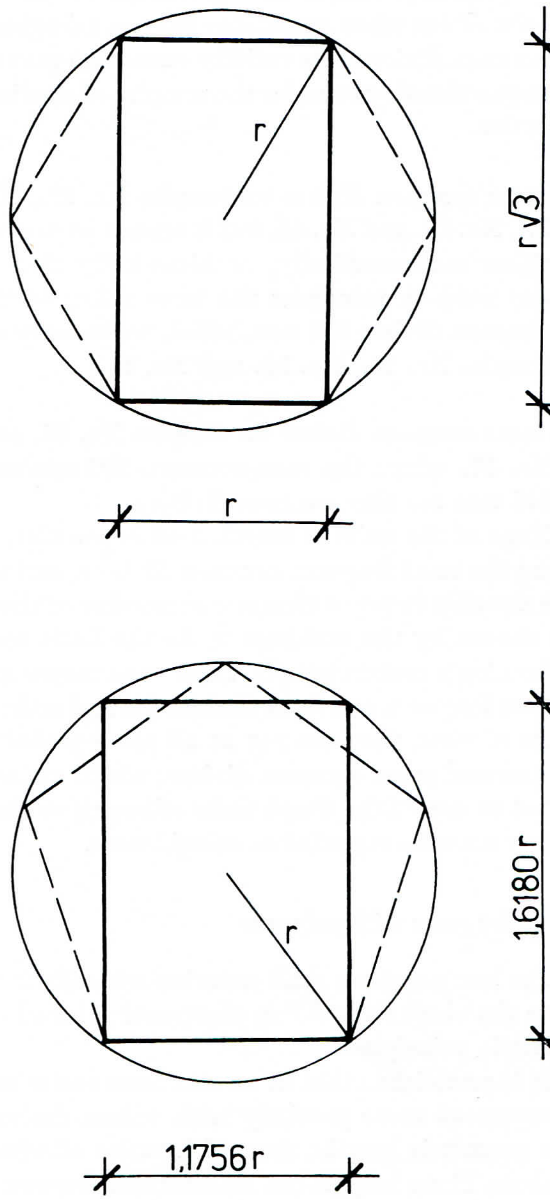


Fig. 2

The ratio flank : width should be 1.73205 in case of a hexagon and 1.3763 in case of a pentagon. The former ratio can be found nowhere; the latter can be detected only on temple No. 45. The same ratio expressed by the side-ratios of the octogon or other (theoretically proved) polygons known in the classical age, can be found nowhere, either.

2. *Front* (see Fig. 3.)

If we suppose that the front can be written in a regular hexagon or octogon then:

	hexagon	octogon
height of ridge : height of order	1.5000	—
width : height of column	—	2.4142

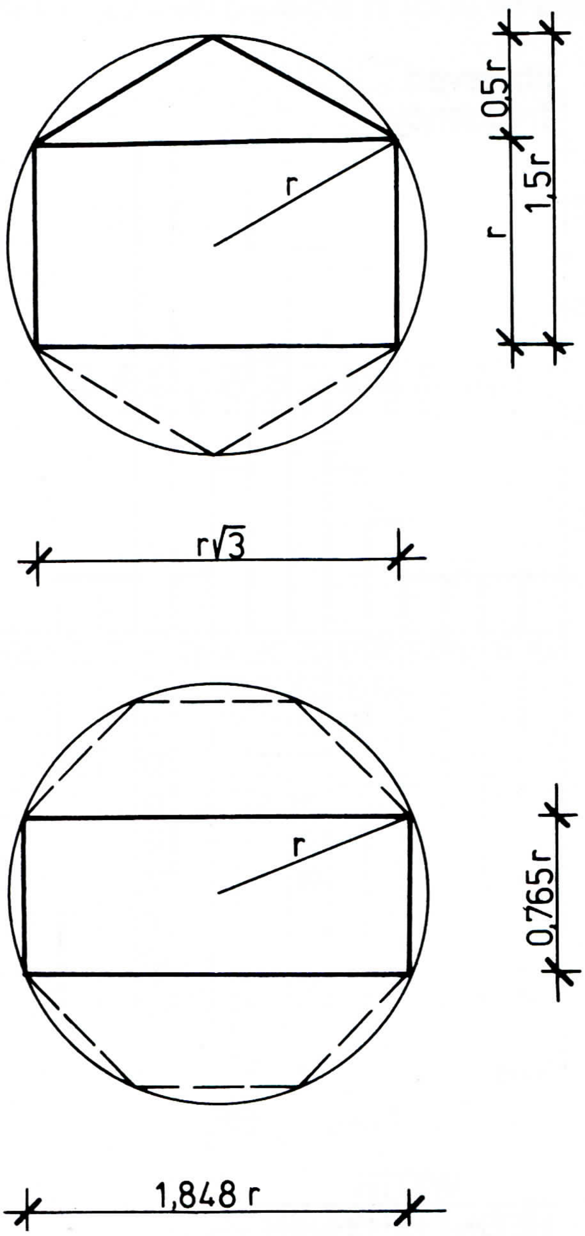


Fig. 3

In the case of a front constructed by a hexagon the ratio height of ridge : height of order of temples Nos 4., 5., 9., 12. and 19. shows a value approximate to the calculated one. Given an octogon the ratio width : height of column of temples Nos 28., 29. and 34. can be proved. The sporadic data make the application of the principle unreliable.

If we consider the stereobate and even the acroterion as ratio factors and mark the points of intersections of the circle and the building elements at random, then of course the polygonal construction of any temple can be demonstrated. However, this exercise does no more than prove its founders' preconception (Moessel, Wolfer-Schulzer).

Thus there is no objective basis for those theories that regard the pentagon or the hexagon drawn in the circle as the principle of temple construction. The statement that the ancient Greeks already built their temples according to the "golden section" as a perfect ratio — can most kindly be called a mistake. But when this unsupported statement is used as a supplement to the myth of a kind of cosmic perfection then we cannot talk of a simple mistake but of an unscientific and antiscientific falsification.

Partly a historical angle, partly the checking of our own perception indicated that we should examine the hypothesis of scales based on geometric principles. In certain cases this procedure may appear feasible, nevertheless, none of its varieties can — due to scarcity of evidence — be regarded as a method of planning.

II. Proof of planning by arithmetic principles

1. Seeking ratios by relating the main measurements

The failure of the methods outlined so far[10] indicates the necessity of returning to other methods just mentioned in the beginning of our paper. Our ideas are as follows: let us suppose that the main measurements of buildings can be related by ratios of whole numbers and let us seek those numbers whose quotient is equal to the quotient of certain component pairs with the smallest difference.

For example, the ratio of flank : width on the Parthenon is: 69,503:30,880 = 2.250745 which is equal to 9:4 = 2.25 quotient value (a very good approximation). So if the planner took 1:4 (30,880: :4 = 7,720) of the width, and took nine times this measurement to get the flank then only a 23 mm constructing imprecision is found. As a starting

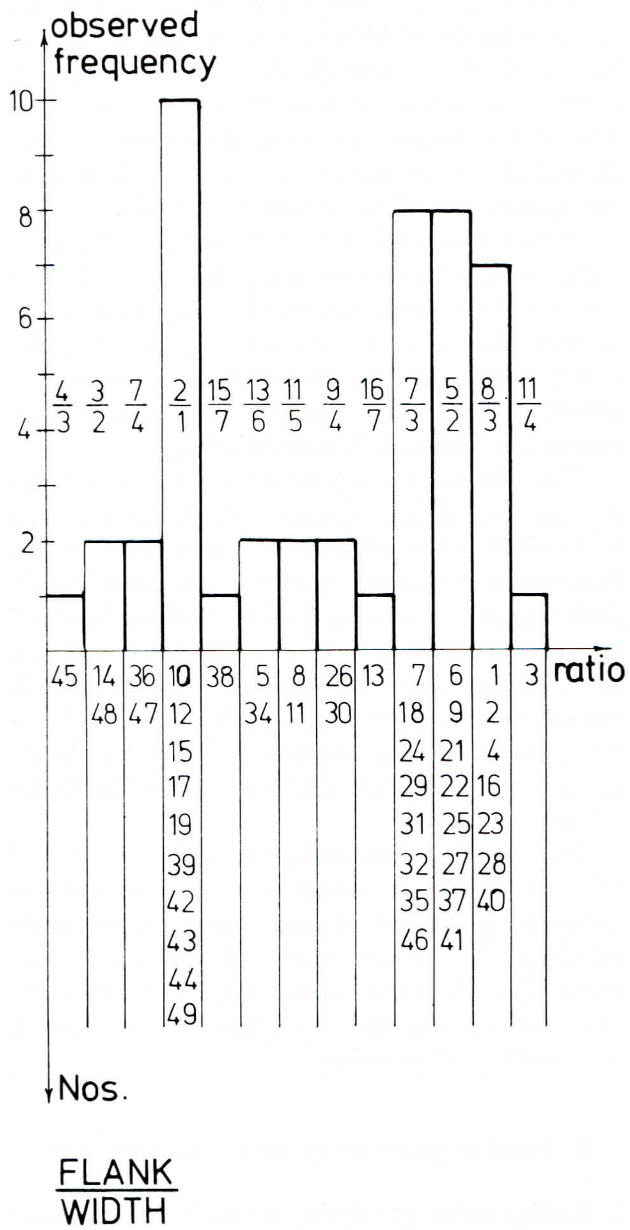


Fig. 4

point we choose the length of the ground plan of the building, take 1:9 of it, multiply that 4 times, and draw that length parallel with the west front pegged out ($69{,}503 \times 4{:}9 = 30{,}890.22$ mm, $30{,}890.22—30{,}880 = 10.22$ mm); the completed temple deviates 10.22 mm from the planned one. Similarly examining the ratio width : height of order $30{,}880{:}13{,}728 = 2.249442$ which also approximately corresponds to $9{:}4 = 2.25$. In the same way the deviation from the gauged value can be calculated 3.56 mm, 8.00 mm respectively. The ratio height of order : height of column is $13{,}728{:}10{,}433 = 1.313307$, approximately equal to the ratio $4{:}3 = 1.3333$. The calculated deviation at the height of column is 209.33 mm and it is 157 mm at the height of order. The ratio width : height of column is ($30{,}880{:}10{,}433 = 2.959839$), approximately equal to the ratio $3{:}1 = 3.00$. The deviation at the front is 139.67 mm and the height of column deviates 419 mm from the measured value.

In some cases, but especially when taking the ratio width : height of column deviation between the gauged and the mathematically precise values is too big. (In the next chapter we shall return to the mathematical testing of ratios made up by approximation. At the same time we also refer to the Appendix I. which provides the approximate and exact values of ratios and the deviation of them for all the 47 buildings.) For a while we will

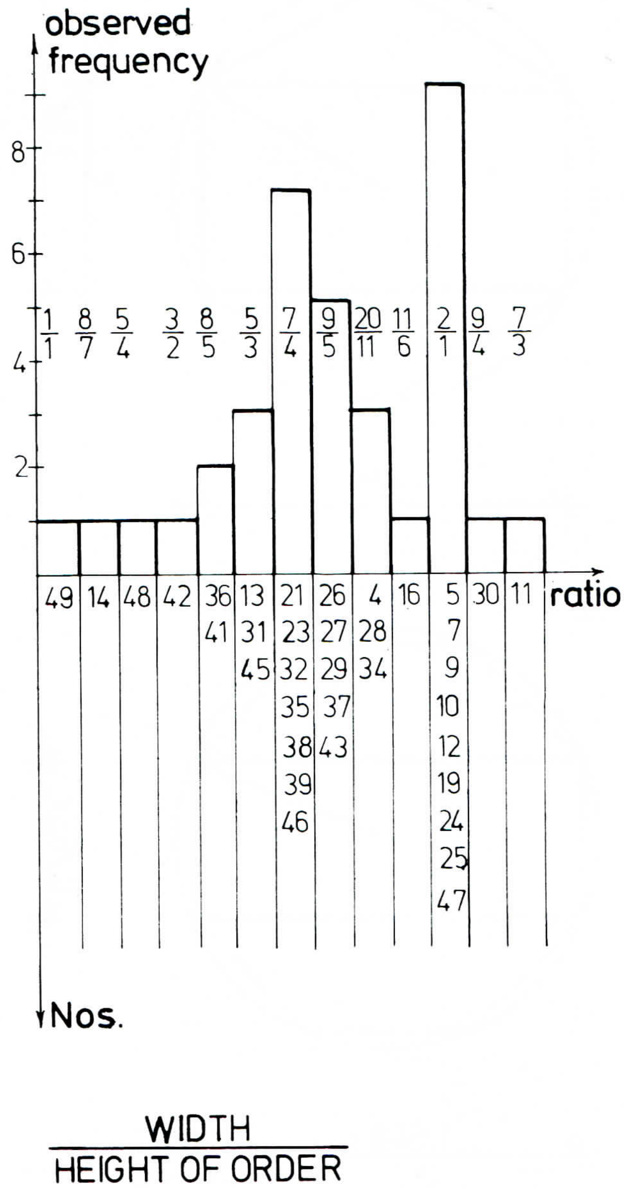


Fig. 5

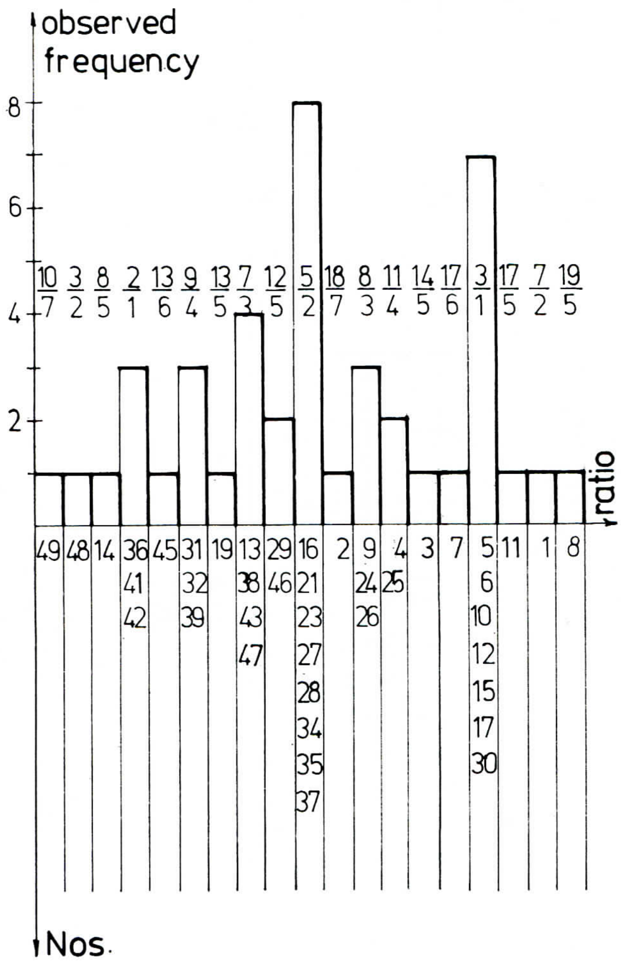
examine the distribution of ratios by approximate values in order to state the "from — to" range and the cultural-historic features of the ratios of the main components with some degree of accuracy.

The ratios of the main components are as follows (see Figs 4—10.). Some notes to them:

Diagram 4: the temples Nos 45., 14., 48. and 36. showing the smallest ratios are not of the peripteral type (see preliminary remarks, point d.) and will not be referred to because they do not form part of the generalization; the temple No. 47. showing the next smallest ratio was built in Pergamon in the Hellenistic age.

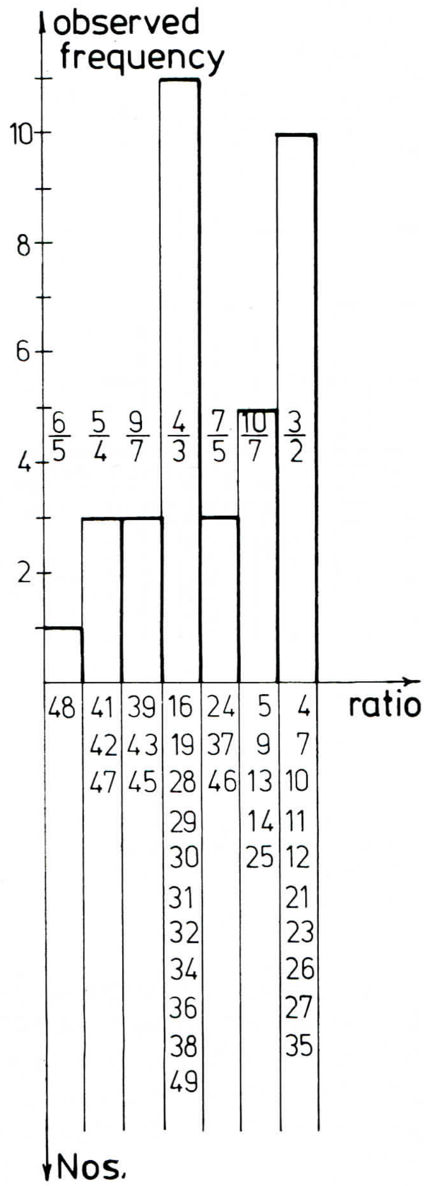
Diagram 5: the temples Nos 49., 14. and 48. showing the smallest ratios — by reason of considerations outlined above — can also be omitted; the same refers to No. 36. in group 8:5; attention is drawn to how small these temples are e.g. Nos. 39., 45. and 46. which show very small ratios.

Diagram 6: temples Nos 49., 48. and 14. showing small ratios can also be omitted here as well as the No. 36. in the next group.



WIDTH
COLUMN HEIGHT

Fig. 6



HEIGHT OF ORDER
COLUMN HEIGHT

Fig. 7

Diagram 7: for the peculiarities of temple No. 48. see above.

Diagram 8: building No. 14. showing the smallest ratio can be omitted as well as No. 36. in the next group; see Fig. 5. in connection with No. 39. having the same ratio.

Analysis of frequency of ratios

2:1, 7:3, 5:2 and 8:3 were the most frequent results when relating the flank to the front. 43 of the temples can be considered for typological generalization, and the ground plan of 35 was measured

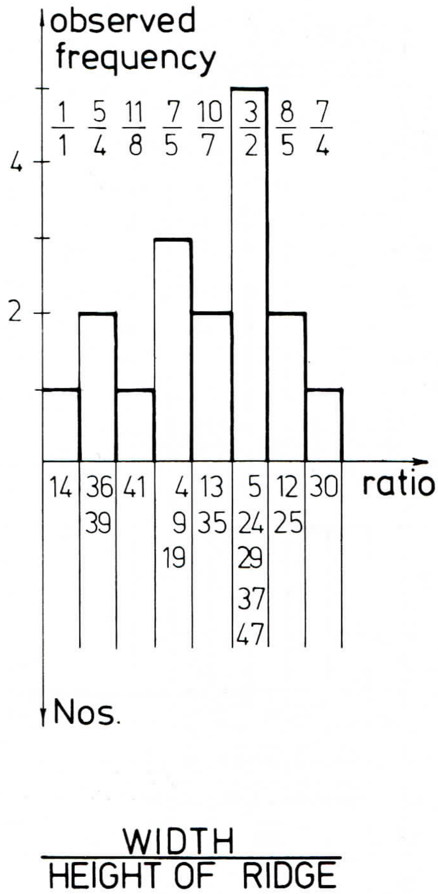


Fig. 8

by one of the 4 ratios. — 41 scales can be found between 2:1 and 8:3 i.e. between 2 and 2.67 in the range; this represents 95.3% of the temples.

The ratio of the width to the height of order was examined on 32 buildings (data from altogether 36 temples are known, 4 of which can be omitted in view of the remark to diagram 3). The results 7:4 and 2:1 are the most general ones found in 16 cases which is exactly half of the temples. — 28 ratios of measurements can be found within these values, that is between 1.67 and 2 in the range, which represents 87.5% of the temples.

The ratio of the width to the height of columns can be examined on 39 buildings (data from 43 are known, but 4 of them can be omitted). The results 5:2 and 3:1 are the most frequent ones found in 15 cases altogether, i.e. with 1/3 of the temples; the other ratio-types are very much scattered. But according to absolute values the "from — to" range is also noticeable here: 29 ratios of measurements can be found between 2.33 and 3, i.e. between 7:3 and 3:1; they occur on 74.4% of the temples.

We have the data from 36 buildings to examine the ratio height of order : height of columns, but one can be omitted. The ratios 4:3 and 3:2 are the most frequent ones, occurring in a total of 21 cases, more than half of the temples. When relating these two components (despite the ratios of measurements examined in the former paragraph) it is clear how few ratio-types, altogether only 6, were used. 29 ratios can be found within the most significant values, i.e. on a very narrow scale between the values 1.33 and 1.5, which includes 82.9% of the temples. But narrow range in a mathematical sense is misleading; as regards the aesthetic effect

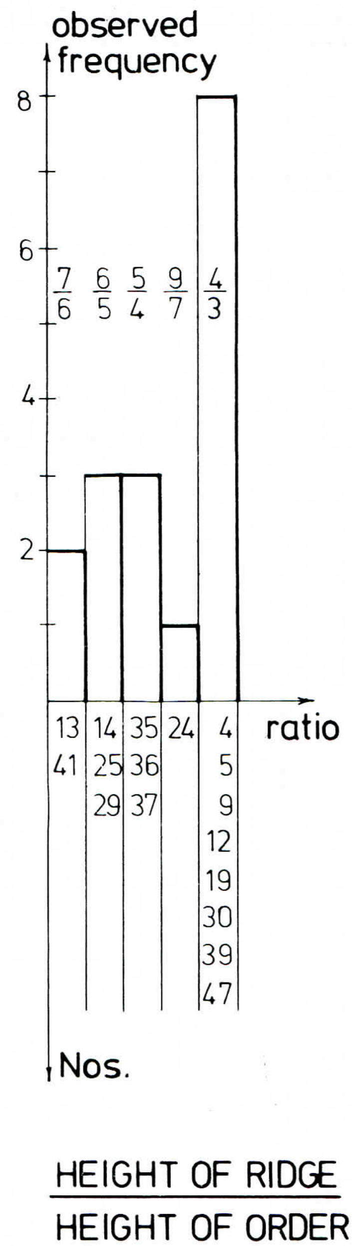


Fig. 9

of the front, it is a very important factor whether the height of the entablature comes to a third or a half of the height of columns!

The height of the ridge is known on only 17 buildings, so the possibilities of describing and generalizing the ratios of measurements with certainty are smaller than in the previous cases. If we leave out buildings Nos 14. and 36. (in view of notes referred to Fig. 8.) 15 temples can be examined for the ratio width : height of the ridge. The results 3:2 and 7:5 are the most frequent ones, occurring in 5 viz. 3 cases in more than the half of the temples. — See the note to Fig. 5. concerning temple No. 39. which gives the smallest ratio. The widest temple is the Parthenon with 8 front columns; it shows, therefore, the biggest ratio. Apart from these two temples the ratio of measurements extend from 1.25 to 1.60 which shows well that these ratios depending on the stocky viz. wide form of the front may have deviated many times from the standard measures. (N.B. This is between the values 1.40 and 1.50 on 10 temples.)

When relating the height of the ridge and the height of order the largest ratio (4:3) was the most frequent one: it characterized almost half of 17 buildings (47%). We can find only 5 relating solutions, but the scattering (between 1.17 and 1.33) is large in this case as well — depending on what process was used in constructing the ratio of the measurements of the tympanum.

When relating the height of the ridge and the height of the columns the ratios 5:3 and 2:1 can be regarded as characteristic (35.3% viz. 23.5%) while on a further 7 temples the other 6 ratios were varied. The scattering of values — between 1.50 and 2.00 — derives from the variant ratios of the height of the entablature and the tympanum.

There are no two such temples where either the actual measurements or the ratios of the main component pairs would be equal. But this variety only seldom meant architectural liberty or a planner's capiice: the majority of buildings demonstrate in absolute value a small range of ratios applied — issue of disciplined phantasy, artistic novelty and respect for traditions as well.

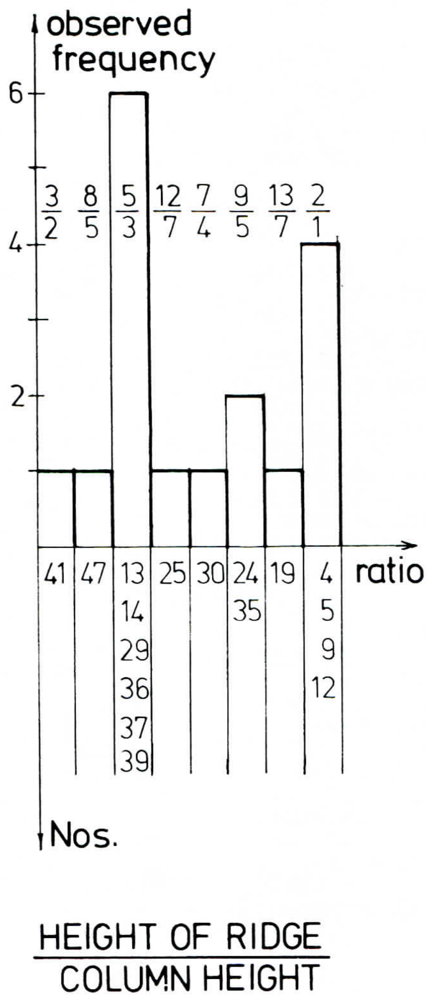


Fig. 10

2. *Examining the ratios from a historic angle*

The statistical-typological analysis is suitable only for a general description of the scales. It seems necessary that we should examine its results from a cultural-historic point of view, that is to discover what particular characteristics can be shown beyond features which are common to architecture formed over great time and geographical distances. It was from a stylistic point of view that we also separated those temples that had 7, 8 or 9 columns on the front instead of the usual 6.

Our research followed two directions: on the one hand we must find out whether a rule can be demonstrated with respect to one scale of the temples belonging to a certain group — e.g. the ratio between the height of the ridge and the height of column —, and on the other hand whether in the group itself there are temples planned according to the same measuring principle, that is such temples on which the ratio of at least two main components is the same.[11]

Because of territorial isolation, we do not consider the temple at Assos (No. 5.); nor on stylistic grounds the temples Nos 14., 36., 45., 48. and 49. For the reasons see "Preliminary remarks", points

a) and d), as well as the remarks to the diagrams in the previous chapters. At certain other temples the material available for examination is diminished by the impossibility of reconstructing the measurements.

We classified the buildings into seven groups. We indicate the groups with Roman numbers, the temples with code numbers:

I. (archaic age, Balkan):	1, 6, 10, 17, 18, 19, 20.
II. (archaic age, South Italy):	2, 3, 4, 7, 9, 13, 15, 16.
III. (classical age, Balkan):	24, 28, 29, 31, 32, 34, 38.
IV. (classical age, South Italy):	21, 22, 23, 25, 26, 27, 35, 37.
V. (7, 8 and 9 front columns):	8, 11, 12, 30.
VI. (hellenism, Balkan):	39, 40, 41, 42, 43, 44.
VII. (hellenism, Near East):	46, 47.

Number of data	Extreme values	Char-acter-istic ratio	Occur-rence in %	Average value
	I. group			
flank : width				
6	2.00—2.67	2:1	50	2.25
width : height of order				
2	2.00			2.00
widht : height of column				
5	2.60—3.50	3:1	60	3.02
h. of order : h. of column				
2	1.33—1.50			1.41
width : h. of ridge				
1	1.40			1.40
h. of ridge : h. of order				
1	1.33			1.33
h. of ridge : h. of column				
1	1.86			1.86
	II. group			
flank : width				
8	2.00—2.75	8:3	37.5	2.48
width : height of order:				
5	1.67—2.00	2:1	40	1.86
width : height of ridge:				
3	1.40—1.43			1.41
width : height of column:				
8	2.33—3.00			2.68
h. of order : h. of column:				
5	1.33—1.50			1.44
h. of ridge : h. of order:				
3	1.17—1.33			1.28
h. of ridge : h. of column:				
3	1.66—2.00			1.89
	III. group			
flank : width:				
7	2.14—2.67	7:3	57.1	2.33
width : h. of order:				
7	1.67—2.00			1.80
width : height of ridge				
3	1.50—1.75			1.58
width : height of column				
7	2.25—2.67			2.41
h. of order : h. of column				
7	1.33—1.40	4:3	85.7	1.34
height of ridge : height of order				
3	1.20—1.33			1.27
height of ridge : height of column				
3	1.67—1.80			1.74
	IV. group			
flank : width				
8	2.25—2.67	5:2	62.5	2.47
width : height of order				
7	1.75—2.00	7:4	42.9	1.81
		9:5	42.9	
width : height of ridge				
3	1.42—1.60			1.51
width : height of column				
7	2.50—2.75	5:2	71.4	2.56
height of order : h. of column				
7	1.40—1.50	3:2	71.4	1.48
height of ridge : height of order				
3	1.20—1.25			1.23
height of ridge : h. of column				
3	1.71—1.80			1.73
	V. group			
flank : width				
4	2.00—2.25			2.16
width : height of order				
3	2.00—2.33			2.19
width : height of ridge				
2	1.60—1.75			1.68
width : height of column				
4	3.00—3.80			3.30
h. of order : h. of column				
3	1.33—1.50			1.44
h. of ridge : h. of order				
2	1.33	4:3		1.33
h. of ridge : h. of column				
2	1.75—2.00			1.88
	VI. group			
flank : width				
6	2.00—2.67	2:1	66.7	2.19
width : h. of order				
4	1.50—1.80			1.66
widt: height of ridge:				
2	1.25—1.37			1.31
width : height of column				
4	2.00—2.33			2.15
h. of order : h. of column				
4	1.25—1.29			1.27
h. of ridge : h. of order				
2	1.17—1.33			1.25
h. of ridge : h. of column				
2	1.50—1.67			1.58
	VII. group			
flank : width				
2	1.75—2.33			2.04
width : height of order				
2	1.75—2.00			1.87
width : height of ridge				
1	1.50	3:2		1.50
width : height of column				
2	2.33—2.40			2.37
h. of order : h. of column				
2	1.25—1.40			1.32
h. of ridge : h. of order				
1	1.33	4:3		1.33
h. of ridge : h. of column				
1	1.60	8:5		1.60

Analysis according to groups:

In group I. the ratios flank : width and width : height of columns serve as sufficient material. The characteristic ratios 2:1 and 3:1 show the safety and simplicity of construction. The same applies (with the single exception of the ratio width : : height of column on temple No. 19.) to the planning of all the temples belonging to this group: they formed the ratios of measurements in such a way as to keep the divisor of the fraction at, or under, 3. — The similarity in the style of temples Nos 10. and 19. is proved by the same 2:1 ratio of flank : width and width : height of order. The similarity between temple No. 10. and the frustrum of No. 17. is demonstrated by an identity in principles of design; the ratios flank : width and width : : height of columns are 2:1 and 3:1, respectively, on both temples. Other factors, e.g. the wide scale of the ratio width : height of columns refer, at the same time, to the fact that in certain cases (e.g. temple No. 1.) essentially different ratios were used.

Homogeneity in group II. is indicated by the ratio flank : width (i.e. the relative frequency of 8:3), width : height of order (2:1 in 40%), and by the narrow range of ratios between height of order and height of columns. Another point of interest is the marked scattering of ratios and the irregularity of certain other ones (e.g. 16:7 or 17:6). Two ratios of measurements on temples Nos 4. and 9. are the same: height of ridge : height of order = 4:3 and height of ridge : height of column = 2:1.

It is noticeable that in group III. the ratio of the height of order and the height of column was planned to be 4:3 in nearly all the buildings and the only different value (7:5) is quite near to the previous ones in absolute number. Four-four sizes of temples Nos 31. and 32. were constructed by nearly the same principles similarly to temples Nos 28. and 34. which differ only in choice of the ratio flank : width.—The narrowness of the range of the ratios also shows the relative compactness of the Balkan architectural style in the classical age.

The classical architecture of South Italy (group IV.) is the most precise and homogeneous considering mathematical features — taking into account the percentage of characteristic ratios as well as the small range of them. The same is shown by that here the identical principle of construction is found most frequently: the temples Nos 27. and 37. have three identical ratios: flank : width, width : : height of column and height of order : height of column; the same is true for temples Nos 21., 23. and 35. in relating width : height of order, width : : height of column and height of order : height of column. The "overlap" of the common features demonstrates the stylistical similarities of these 5 temples. The other three buildings do not show significant differences either.

We put the temples with 7, 8 or 9 front-columns in the group V. in order to compare their probable peculiarities to the ratios of the others. The largeness of the ratio (quotient) between the width and the vertical elements is most remarkable: these temples have a much wider front than those of with 6 front columns. Two data are equal from the 4 data with respect to flank : width (Nos 8. and 11.) and similarly 2 from the 3—3 data with respect to width : height of column and height of order : height of column (Nos 12. and 30., 11. and 12. respectively). Temples Nos 12. and 30. show the same principles of construction where both the height of ridge and the height of order are equal (4:3), too.

The group VI. includes the temples of the Hellenistic Balkan. They are characterized by the majority of ratios flank : width 2:1 as well as by small range of ratios height of order : height of column. In two cases we found the same principle of construction on two-two component pairs: temples Nos 39. and 43. have the same ratios flank : : width and height of order : height of columns, and temples Nos 41. and 42. have the same ratios width : height of column, and height of order : height of column. Compared to these the scattering and uniqueness of ratios flank : width and width:height of order on other buildings are well noticeable, a fact that can be called a symptom of loosening of architectural principles of the classic age.

All we can state about the two temples included in the group VII. is that they were built by dissimilar principles.

A comparative analysis (according to component-pairs):

When grouping the temples according to age and territory we find that we cannot attribute their planning to *one* rule even within small historic groups. As regards the scales, the temples of South Italy of the classic age proved to be the most compact ones, in which many corresponding component pairs are the same and their differences are of a smaller range than those in the other groups.

In a certain group — especially when the number of temples included and the reliable data are small — even one "irregular" building (which essentially differs from the others) significantly influences the average value of scales. In this analysis however, where we compare the characteristics of individual groups, these irregular examples naturally have a place as well.

Extreme values of ratio flank : width are as follows: 2.04 (group VII.) and 2.48 (group II.). We cannot, however, draw a linear historic line even then when we omit the group VII. which has too little data for proof. It seems sure that in South Italy the expanded rectangular shaped ground plan was preferred and in the Hellenistic age the ratio approached was 2:1.

Taking the ratio of width and height of order the temples in group V. are to be the most expanded i.e. those which have 7, 8 and 9 front columns (2.19).

When relating the width and the height of column the gradual, but not even, diminishing of the archaic ratio of 3.02 can be observed. Projections jutting out are noticeable at temples with 7, 8 or 9 front columns (3.30) which is just such a stylistic feature that was found in the case of the previous ratio and is naturally a complement to it.

There is no essential change in the ratio of the height of order and the height of columns during the archaic and classic ages but this ratio diminishes to some extent on Hellenistic temples (the columns grew compared to the height of order: the entablatures became lower).

We know the height of ridge of 17 temples with approximate precision, but there are only two such historic groups (South Italy of the archaic and classic ages: groups II. and IV.) from which we can draw a concluding comparison from the relationship of the main components at 3—3 temples. The average ratio of width and the height of the ridge has grown: in the classic age wider temples were planned. The average ratio of the height of order and the height of column almost exactly corresponds in the two periods but the ratio of the height of ridge and the two other vertical measurements diminishes significantly — in other words: the tympana were planned flatter in the classical age than earlier.

When relating the front and the height of ridge the same characteristic tendency i.e. expansion is found elsewhere too till the end of the classical age, but in Hellenism — so far as we can tell from the limited data — the planning style became characteristically stumpier again. The ratio of the height of ridge and the height of column similarly diminishes on the Hellenistic temples (as already mentioned, the entablature was planned lower), but the ratio of the height of ridge and the height of order did not change significantly (the proportions of the tympana were not modified in a noticeable way).

In some cases we succeded in separating the features characterizing certain groups and thus in differentiating our statements concerning the whole Doric style. However, despite cultural-historic or structural peculiarities the range of ratios is small over-all from which only a few "irregular" buildings differ essentially. We also found groups of temples planned by identical principles of construction but neither the general principles of the style nor the similarities led to a mechanic imitation or a dogmatic inflexibility.

3. *Checking the hypothesis "scales = ratios of whole numbers"*

In our examination so far we indicated the ratios of the main components with approximate precision. But when investigating the ratio width : : height of column at the Parthenon we already indicated that the actual datum differs from the value of the quotient of whole numbers standing nearest to it, to a larger extent than could be allowed. Let us illustrate this with the scales of another temple:

The ratio of the front and the height of order on the temple of Zeus at Olympia (No. 24.) is 1.9076 which we rounded to 2.00 with a seemingly insignificant modification. If we suppose that the main component pairs were related by whole numbers then 19:10 would express this ratio far more precisely. However this is in practice improbable because it would suppose that the front was divided into 19 units from among which the planner chose 10 to construct the height of order. This solution assumes a clumsiness from the technical and logical as well as aesthetic point of view. That is why we have used — not only here but generally — those fractions of whole numbers, which seem to be more probable as a means of relating components.

However comfortable this method may be, the illustration above is warning of using it in a rough-and-ready way. In the interests of proof (the possibility of generalization) or the correction of our hypothesis we have to return to the principle of size tolerance that was already used in the search for a module, in other words we must presume that the maximum difference between the gauged measurements and the planned (calculated) ones can be ±200 mm for the flank, ±100 mm for the width, ±50 mm for the height of column, ±100 mm for the height of order and ±150 mm for the height of ridge.

Our method will be as follows: fractions of whole numbers which can be proved within the presumed size tolerance can be accepted as means of relating sizes.

On this basis let us examine the modification of the 1.9076 quotient of width : height of order to 2 at temple No. 24. — For this modification to meet our requirement, the maximum increase within the size tolerance of the dividend (27.78 instead of 27.68) and the maximum decrease of the divisor (14.41 instead of 14.51) would be sufficient. But 27.78:14.41 = 1.9278 and this is still far from 2, the value of approximation which we accepted for the time being. So that the ratio width : height of order now becomes 2:1 and presuming that the architects were equally mistaken in the building of both the components then the required correction would be plus 38 cm at the width and minus 38 cm at the height of order, as 28.06:14.03 = 2:1. However it is unlikely that such a great error was made in building such an excellent temple: we must exclude the possibility that the front was built with an error of 38 cm (too small) and that the height of order was built with an error of 38 cm (too large). — According to the same principle we have to select the other ratios, too.

But what should the general criterion of the selection expressed in figures be?

We can reach this criterion by constructing models: according to some frequent ratios we shall construct the main components and within the maximum ± deviation we examine the difference between the quotients of the plan and those achieved.

Model I. Let us make the height of order 10 m, the ratio with:height of order 2 : 1=2, the flank: width 5 : 2=2.5 — so the width is 20 m and the flank is 50 m.

a) variation	difference
width : height of order: 20.10:9.90 = 2.020	+0.020
flank : width: 50.20:19.90 = 2.523	+0.023
b) variation	difference
width : height of order: 19.90:10.10 = 1.970	−0.030
lank : width: 49.80:20 .10 = 2.477	−0.023

Model II.: The height of order is 12 m, the ratio width : height of order is 7:4=1.75, flank : width 7:3 = 2.333 — so the width is 21 m, the flank is 48.993 m.

a) variation	difference
width : height of order: 21.10:11.90 = 1.756	+0.006
flank : width: 49.113:20.90 = 2.353	+0.020
b) variation	difference
width : height of order: 20.90:12.10 = 1.727	−0.023
flank : width: 48.773:21.10 = 2.324	−0.009

After having examined these temples with different measurements and scales it is clear that the maximum size tolerance results in approximately 0.03 deviation of the quotients gauged from those prescribed in the plan. We of course cannot reach an absolute conclusion (the examination of smaller elements or larger ratios may show a larger deviation) but if we select out strictly the ratios achieved by approximation we can justifiably choose the deviation calculated on the models as a starting point: we regard those ratios which stay within this limit as the proof of our hypothesis.

As regards the material for examination some enlargement seems to be justified. We must not be satisfied with the data of the main components but we shall draw into our calculation also the measurement of the axial spacing (either as a dividend or a divisor) of the front columns.

As a complement we shall put into our chart those data by DINSMOOR which do not show more than 0.03 deviation from the value of fractions from whole numbers. The columns of the chart are as follows: flank : width (I), width : height of order (II), width : height of column (III), height of order : height of column (IV), axial spacing : diameter of the column (V), height of column : axial spacing (VI), height of order : axial spacing (VII), width : height of ridge (VIII), height of ridge : height of order (IX), height of ridge : height of column (X). —See the data on p. 304.

Well, let us sum up the ratios accordingly to groups which could be expressed by fractions from whole numbers:

I. flank : width	from 47 cases 31 = 65.9%
II. width : height of order	from 36 cases 19 = 52.8%
III. width : height of column	from 43 cases 26 = 60.5%
IV. h. of order : h. of column	from 36 cases 27 = 75.0%
V. axial spacing : column diameter	from 47 cases 21 = 44.7%
VI. h. of column : axial spacing	from 43 cases 15 = 34.9%
VII. h. of order : axial spacing	from 36 cases 15 = 41.7%
VIII. width : height of ridge	from 17 cases 14 = 82.3%
IX. h. of ridge : h. of order	from 17 cases 16 = 94.1%
X. h. of ridge : h. of column	from 17 cases 14 = 82.3%

No.	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
1.	8:3	—		—	11:4		—	—	—	—
2.	8:3	—		—			—	—	—	—
3.	11:4	—	14:5	—	11:5		—	—	—	—
4.	8:3		11:4	3:2			3:1	7:5	4:3	2:1
5.	13:6			10:7				3:2	4:3	
6.	5:2	—		—	7:3	9:5	—	—	—	—
7.	7:3		17:6	3:2				—	—	—
8.	11:5	—	19:5	—	2:1	9:4		—	—	—
9.			8:3	10:7	5:2			7:5	4:3	
10.								—	—	—
11.	11:5	7:3	17:5		11:5	9:4	13:4	—	—	—
12.					2:1			8:5	4:3	
13.	16:7	5:3		10:7		7:3	10:3	10:7	7:6	5:3
14.			8:5	10:7			8:3	6:5		5:3
15.		—	3:1	—		7:4	—	—	—	—
16.	8:3	11:6	5:2			3:1		—	—	—
17.		—		—			—	—	—	—
18.	7:3	—	—	—	5:2	—	—	—	—	—
19.					8:3	2:1	11:4	7:5	4:3	13:7
20.		******	*****		******	******			******	******
21.	5:2	7:4	5:2					—	—	—
22.	5:2	—	—	—	9:4	—	—	—	—	—
23.	8:3	7:4	5:2					—	—	—
24.	7:3		8:3	7:4	7:3	2:1		3:2	9:7	9:5
25.			11:4	10:7		2:1		8:5	6:5	12:7
26.	9:4	9:5	8:3		9:4			—	—	—
27.	5:2	9:5					3:1	—	—	—
28.	8:3			4:3	7:3	11:5		—	—	—
29.	7:3		12:5	4:3		11:5	3:1		6:5	5:3
30.	9:4	9:4		4:3	9:4			7:4	4:3	7:4
31.	7:3	5:3	9:4	4:3				—	—	—
32.	7:3	7:4		4:3		7:3		—	—	—
33.		******	******		*****	******			******	******
34.				4:3	8:3			—	—	—
35.	7:3		5:2	3:2	11:5		3:1	10:7	5:4	9:5
36.	7:4	8:5		4:3			10:3	5:4	5:4	5:3
37.	5:2	9:5		7:5	9:4		3:1	—	5:4	5:3
38.	15:7	7:4	7:3	4:3			3:1	—	—	—
39.		7:4	9:4	9:7				5:4	4:3	5:3
40.	8:3	—	—	—		—	—	—	—	—
41.	5:2	8:5	2:1	5:4	7:3			11:8	4:3	3:2
42.		3:2		5:4		11:4		—	—	—
43.		9:5	7:3	9:7		9:4		—	—	—
44.		—	—	—		—		—	—	—
45.		5:3	13:6	9:7	5:2			—	—	—
46.			12:5	7:5	9:4			—	—	—
47.	7:4		7:3	5:4		11:5	11:4		4:3	8:5
48.	3:2	5:4	3:2	6:5	13:4	11:5	8:3	—	—	—
49.			10:7	4:3			3:1	—	—	—

Remarks:

1. The data of certain columns (e.g. IV, VI and VII) are mathematically connected — though this does not mean that if data from two of them can be expressed by fractions from whole numbers then it is necessarily true for the third one as well. Let us assume e.g. that the ratio of the height of order (a) and the height of column (b) is 4:3 and the ratio of the height of column (b) and the axial spacing (c) is 11:5, then the ratio of the height of order and the axial spacing is a:c = 44:15. This fraction (in absolute value = 2.933) cannot be reduced, and its value ought to be enlarged by 0.067 in order to identify it with the ratio 3:1 — so it does not square with the hypothetical requirement of the "good ratio". This can be seen e.g. at temple No. 28.

Generally speaking: if two out of these three sets of data can be considered "good ratios", it is enough to describe precisely the planning of the front.

If the temple was built with minimum enlargement of the height of order and/or minimum decrease of the axial spacing — so in order to approximate the quotient 2.933 to 3 (cf. temple No. 30) — this "loosening" did not affect the need for arithmetic precision. On the contrary it proves that relating whole numbers was considered as a basic principle of planning. To us it shows that the mathematic strictness with which we selected the "good ratios" was slightly exaggerated.

2. The error made in building small components (e.g. the difference of a few centimetres in the intercolumnations and the axial spacings) may spoil the mathematically required precision. Let us presume e.g. that according to the plan the diameter of the column is 1.20 m and the axial spacing is 2.70 m (4:9), so the intercolumniation would be 1.50 m. If however the gauged datum of the latter one is only 1.46, then the value of the ratio axial spacing : column diameter (2.66:1.20) would be only 2.217. If we vary this value by the ± 0.03 allowed in the hypothesis we cannot identify it with a ratio of 2.25 (= 9:4) but at most with the value 2.2 (= 11:5).

In other cases we can even qualify — or could qualify — such a large deviation as intolerable. It seems that the schematic use of theoretically justified mathematical strictness results in exaggerated selection.

3. The same statement applies to the ratio of large components, first of all to the ratio flank : :width. It is quite improbable that, while the scales on the front of a certain temple were planned precisely, those of its ground-plan should have come about by accident (Nos 9 and 25). Instead we should rather think that imprecision was caused by a building error: if the builders deviated only 3—5 degrees from the right angle required at the meeting of the front and the flank then not only the measurements of the front and the backfaçade differed but also the ratio width : flank overstep our tolerances, especially in the case of larger buildings — consequently the ratio must be qualified as wrong.

4. Imprecisions that are insignificant in themselves may cause a deviation which is mathematically inadmissible. E.g. when relating the front, height of order and the height of column of the Parthenon (No. 30.) we found the following: width : :height of order = $2.2491 \approx 2.25 = 9:4$, height of order : height of column = $1.3158 \approx 1.333 = 4:3$. A logical result would be that the ratio width: :height of column = 3:1, however in reality it is 2.9598, which we have already rejected. The deviation is caused by the fact that the height of column is slightly larger than 3:4 (0.759) of the height of order and as the ratio between the width and the height of order is below 2.25, the surplus of the new divisor (height of column), which exceeds the ideal, results in a significant decrease in the new quotient.

5. The components of the front were always built more precisely than those of the flanks. Apart from the subjective faults, certain static requirements and stylistic demands also reduce the possibility of a mathematical exactness: the contractions and the thickening of external columns make relationships with the axial spacing uncertain. Moreover, the horizontal curvature (as the lower and upper lines are not perfectly parallel) produces in the height of column slight differences in absolute value, which must affect the exactness of ratios, too.

6. In the first chapter of our paper where we examined the thesis "module = lower diameter of columns", we demonstrated that this component could not serve as a basis for planning larger ones. There is namely no proof for the principle use of whole-number multiplication of either the diameter or the radius. Compared with this fact axial spacing (= diameter of column + intercolumniation) occurs frequently as a means of measuring components, and what is more it is

the ratio of axial spacing and diameter of columns at times perfectly precise. The two statements are not however contradictory, for this latter ratio can be expressed as whole number only in a few cases (temples Nos 8. and 12.).

7. The reality of certain "odd ratios" can be doubted (e.g. 19:5 or 17:6) either because of their isolation or because their absolute values do not differ much from other, frequent and mainly simple ratios. The principle of size tolerance indicates however that we should stick to these even being aware of exaggeration and that we should not be tempted to write 4:1 instead of 19:5 and 3:1 instead of 17:6.

8. We found temples that were remarkably imprecise and others that were in every ratio precise, as well. The former group is represented by the temple of Athena (No. 10.) built by the Peisitratids between 529 and 515 B. C., where no scale can be regarded precise form the point of the size tolerance principle. This fact can be explained by rough building in the archaic age (we must not ever forget that practice of building had preceded mathematically conscious planning) and on the other hand it can also be explained by vagueness in reconstruction. — The second group can be illustrated by the temple of Dionysus at Pergamon (No. 48.) built in 170 B. C., 7 ratios of which are perfectly precise. In the case of this temple, peculiar in other ratios also (see remarks to the diagrams), the smallness of the measurements could have eased the builder's job: the danger of spoiling it was smaller than with much larger buildings. — Apart from these extremities the tendency to require precise ratios is clear: it cannot be accidental that at the temple of Apollo at Selinus, built as early as between 520 and 460 B. C., six out of seven ratios filled this requirement.

Conclusions:

If we define the scales mathematically strictly and even with evident exaggeration, their numerical majority proves that the main components were planned by relating whole numbers. First of all the precise ratios of the height of order and the height of column (75%), of flank and width (65.9%), of width and height of column (60.5%) prove the correctness of our hypothesis[12] — even if the good reasons for rounding-up the percentages are not considered.

We do not say that in every case we can explain the deviation between data gauged and measurements considered mathematically correct. We have not even attempted to test our hypothesis outside the main components (e.g. to test the ratio of the length of the architrave to the horizontal of the tympanon or that of the height of the architrave to the frieze). However we think that the results are convincing for demonstrating the general methods of planning.

Throughout we have adopted the concepts of tolerance and mathematical accuracy simultaneously, as two instruments with which to check our statements jointly and from two different starting points. Neither the one nor the other can be disregarded if we wish to arrive at reliable generalizations:

If we dismiss the concepts of tolerance, i.e. we rigidly equate gauged data and their ratios with those the original plan is supposed to have contained, then we are destined to arrive mostly at negative results (perfectly accurate ratios are extremely rare, therefore the logical conclusion would be that there is no consistency in the planning of temples). If on the other hand we refuse to seek mathematical accuracy, then the most we can do is to deliver some high-faluting eulogy of the beauty of Greek temples, denying Greek culture its inherent rationalism.

The adequacy of our directions and methods of analysis must be verified by the statements already available and reinforced in the next chapter.

Provisionally, let us draw some conclusions:

1. When seeking the main components and their ratios, we chose the measurement of the width as a starting point. In cases where we found that a frontal component could not be related (by whole numbers) to the width, though it was in such a ratio to some other part of the building, we naturally expanded the circle where ratios were to be established.

2. Practice has proved that size tolerance is justified in our case, though at times the ± range or the method of establishing a "false result" turned out to be too rigid. Such an error was corrected e.g. in establishing the ratio between the width and height of columns of the Parthenon, the idea of correction being supported by another consideration: when finding that the ratio 2.959839 could only be rounded up to 3 at the cost of 139.67 mm and 419 mm deviation on the width and on height of column respectively, then we regarded

one of the two measurements identical with the actually gauged one and supposed the other to be different from the value established by archeologists.

This method, however, would be rather mechanical. In case of differences of such magnitudes we must accept that both measurements might differ from the values that we deem accurate (i.e. believed to be factors in ratios). Therefore, we can correct both the dividend and the divisor by a few hundredth — maximum 0.05 — upward or downward.[13] If, in the case of the ratio 4:3 between the Parthenon's height of order and height of column, we add 0.01 to both factors, the resulting 4.01:3.01 ratio (1.3323) gives a good approximation of the ratio of gauged data: the difference will in the first dimension be only 9.96 mm, in the second 13.10 mm. Similarly, we may correct the ratio width : height of column to 2.99:1.01. The resulting values of deviation will be 1.96 mm and 5.81 mm. Total correspondence of values (i.e. differences strictly within the tolerance granted) did not occur in every case, but there is a theoretical possibility of it. The Appendix contains the corrected values, intended to bring actual ratios closer to the ideal ones, for every building examined.

3. Since we only accepted a tolerance of 0.03 for "good scales", we had to dismiss ratios like 44:15 = 2.933. Knowing the measurements of the building in question, it can be demonstrated that rounding 44 to 45 means a deviation only by 2.5 % i.e. 22 mm. On this account, we think it is possible and necessary to continue our studies along this new lines in another essay: instead of analysing ratios of measurements, we shall have to establish the magnitude of differences — both in absolute terms and in per cent rates — which is a result

No.	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
3.	11:4		14:5		11:5					
4.	8:3			3:2			3:1		4:3	2:1
8.	11:5		19:5							
9.			8:3						4:3	
11.	11:5		17:5		11:5	9:4	13:4			
13.	16:7	5:3		10:7		7:3	10:3	10:7	7:6	5:3
14.			8:5				8:3			5:3
19.					8:3				4:3	
21.	5:2		5:2							
24.	7:3		8:3	7:4	7:3				9:7	9:5
25.				10:7				8:5	6:5	12:7
26.	9:4	9:5			9:4					
28.	8:3			4:3	7:3					
29.	7:3		12:5	4:3		11:5			6:5	5:3
30.	9:4	9:4		4:3	9:4			7:4	4:3	7:4
31.	7:3	5:3		4:3						
32.	7:3	7:4		4:3		7:3				
34.				4:3	8:3					
35.	7:3		5:2	3:2	11:5		3:1		5:4	9:5
36.	7:4			4:3			10:3	5:4	5:4	5:3
37.		9:5		7:5	9:4			6:4	5:4	5:3
38.		7:4	7:3	4:3						
39.		7:4	9:4	9:7				5:4	4:3	5:3
41.	5:2			5:4						
42.				5:4		11:4				
43.		9:5		9:7		9:4				
45.		5:3			5:2					
46.			12:5	7:5						
47.	7:4		7:3	5:4		11:5	11:4		4:3	8:5
48.	3:2	5:4	3:2	6:5	13:4	11:5				

Notes:

a) almost two-thirds of buildings have ratios forming some kind of systems, the types of which could be chosen by the planner at his discretion;

b) it was in line with our hypothesis and method that we refused to round even slightly the absolute values of ratios, thereby increasing the amount of positive evidences, as would have been the case in

12. 2:1 2:1 3:1 3:2 2:1
19. 2:1 2:1 8:3 4:3 8:3 2:1;

c) in certain instances it might give rise to doubts about the existence of a system of ratios that there are only two scales to support it, or that measurements of statically and aesthetically disparate components have been related. We are of the opinion that even such a sceptical attitude would leave unharmed our conclusive demonstration as to the existence of systems of ratios with the great majority of temples.

of inaccurate implementation of design rather than that of neglecting principles of design supposedly true.

4. Our findings indicate that the principle of tolerance was justified, but that its application requires certain flexibility on account of the considerable differences in the measurements of buildings. What this implies regarding the module theory is that by an expansion in hypothetical tolerance (i.e. ± 300 mm, in place of the normal ± 200 mm, tolerated on the flank of a big temple, etc.) the number of common divisors of components will be multiplied, which in turn would only reduce the probability of one of them being *the* common divisor, i.e. the module.

4. Systems of ratios on the buildings examined

The principle of planning consisted in establishing whole-number ratios between components, chiefly main components, but the ratios thus obtained were treated liberally within a "from... to" range in the Doric style. Further, it was not a law to be obeyed that one and the same key number should have been applied throughout the building as a factor in ratios: if two dimensions were set e.g. at 4:9, another two might be set at 1:2 or 5:11.

The support given to this hypothesis by our findings listed above is only reinforced by the fact that the principal ratios on several temples form systems of ratios. These systems have the following types:

a) a:b=b:c (e.g. flank : width = width : height of order),

b) a:b = c:d (e.g. flank : width = axial spacing : column diameter),

c) the type that repeats either the dividend or the divisor thereby showing the conscious choice of dimensions for components (e.g. 3:1 and 3:2; or 5:3 and 7:3).

The plate above (p. 307) gives a list of temples whose components form systems of ratios. (For the numbering of columns, see previous description.)

III. Mathematics and Aesthetics

After examining current theories in the literature, we have found that neither Vitruvius' module theory (i.e. that components were measured in proportion to the radius of the columns) nor the assumption "module = some practical measurement of that age" are borne out by facts. We have presented data refuting the idea of explaining the design of ground plans or fronts of Greek temples through the ratios of circumscircuses, too. Moreover we have proved that theories that attribute to the Greeks the mysticism of building-symbolism — chiefly by the allusion to the "golden section" — are erroneous.

We have shown that there is no common divisor (module) in buildings that crops up so frequently as to make it conspicuous. Thus if any of the common divisors pertaining to the measurements of a particular temple could function as a module, then one or another of them would have to be found repeated in other buildings to mark its special status. However no such universal unit has been found in spite of considerable tolerance in values.

On the positive side, we have established that the ratios of components (especially main components) can — with a certain measure of approximation — be expressed by ratios of whole numbers, where — simply enough — the divisor is never greater than 8. We have adhered to strict principles in postulating and adopting "good ratios", nevertheless, it has been demonstrated for the majority of buildings that the basic principle underlying their ratios was division and multiplication by whole numbers.

Our method of analysis naturally contains a subjective element, namely the substantiation of tolerance, a device that may lead to schematic generalization and inaccuracy if applied without discretion. Therefore the results we have obtained should be used also as a basis for a new method of analysis: by introducing insignificant (in absolute terms) modifications in the scales we should explore deviations that can be explained by inaccuracy in the implementation of principles of design supposed to be clearly ascertained.

That some arithmetical system of relating was used in almost two thirds of the temples examined proves our hypothesis that components were planned by ratios of whole numbers, and that it was a principle of conscious planning to develop ratios *(logos)* into a system of proportionality *(analogy, symmetry)*.[14]

All this is in harmony with the principal and most general theorem of aesthetics in the antiquity, equally applied to music and the visual arts: that accuracy of ratios is a precondition to beauty.

A large number of examples eloquently testify to the mathematical element behind the planning of major components. This is proved, e.g. on the front of the temple of Asclepios at Epidaurus, where the sum of the 6 column diameters and that of the 5 intercolumniations are exactly the same.

The general basic principle was implemented through a variety of ratios without any of them used exclusively or so frequently as to turn theory into dogma: artistic practice was far more lively and variegated than the basically normative aesthetics of the Greeks and Romans (which froze into a collection of rules with Vitruvius).

Care must be taken to see the limits of using mathematics in handling ratios, though. It must be stressed that we have only examined the ratios of *main components*. A similar approach to secondary or decorative elements is but rarely justified. For example:

If, in a temple with 6 columns at the front, there are 13 columns on the flank (as prescribed by the classical canon) and the ratio of axial spacing and column diameter is, say, 2.25:1 on the front, then either this ratio or any other expressible in a fraction of whole numbers is impossible to obtain on the flanks.

Or: we have inadequate informations about the apportioning of the height of ridge, i.e. the ratios between the respective heights of the architrave, the frieze and the tympanum; again, neither is it possible to discern the mathematical principle in the ratio between the vertical and the horizontal line of the tympanum.[15] We arrive at similar conclusions when comparing column diameters to the main components: instances in which the quotient is exactly or approximately a whole number are sporadic (statistically accidental).

Furthermore: differences between the horizontal measurements of the front, the architrave and the tympanon can be stated but cannot be expressed in a mathematical formula. The same holds true of the "triglyph conflict", the unresolved task of constructing a formula for the width of the architrave, the difference between the stylobate and the length of the architrave, the side of the abacus square and the upper diameter of the column.[16]

Another example: there is no apparent universal principle in the entasis of columns, either in the design of their inclination, or — consequently — in the ratio between the upper and lower diameter.

Higher mathematics is able to describe almost any curve with one formula or another. However, it would be totally unrealistic to suppose that e.g. the curve of the echinus was constructed according to the formula of the hyperbola, or that the ancient architect applied the formula of the parabola or some kind of quadratic equation in other cases (e.g. when designing the flutes or the horizontal curves of the front) — simply because contemporary mathematicians had not yet discovered these formulae. Along such lines, it would be possible (both for the Greeks and primitive peoples) to "prove" that illiterate potters, hardly able to count, shaped their vessels by imitating the shadow of a circle lit at an oblique angle or by applying the formulas of the hyperbola and the parabola, not to speak of bakers and their loaves.

In many cases we have been unable to demonstrate a mathematical principle beyond doubt even for ratios between main componentes. The conscious choice of calculated ratios, as a principle of planning, does not rule out the recognition that the architect was in certain cases guided by practical necessity or by the need of creating something pleasing, one that is often impossible to express in terms of mathematics.

It is understood that large, statically and aesthetically complex buildings cannot be built without mathematical consciousness. But mathematics was only a tool (even in the case of major scales) to create rational beauty — the Greek aesthetic ideal — and not identical with aesthetic beauty itself. Aesthetic pleasure is influenced partly by objective factors (like the relationship between building and environment) and partly by historical and individual factors always present in any object-subject relationship. The most perfect truth about mathematical ratios is only a half-truth as regards aesthetic effect.

To illustrate this, let us recall some optical laws:

Even though the scales on the fronts — the most aesthetically significant planes on buildings— of two temples are identical, the small one will evoke an optical effect and a mental reaction entirely different from those created by the large one with a mathematically similarly designed front.

The contrast of horizontal and vertical elements (e.g. the ratio between the thickness and height of a column), differences in dimensions of elements of identical direction (narrow spacing not only enhances the illusion of solidity and massiveness but also brings the visual image of the oblong of the order closer to a square), the entablature which arrests the eye travelling upwards the building, the contradistinction between the tympanon and the columns (if two temples of equal height differ in that the one has considerably higher and thinner columns than the other, the former will itself look taller), the accelerating effect on the vision of the flutes, the arresting effect on the eye and on the mind by the frieze, the visual pleasure generated by ornamentation and colouring — all these contribute to presenting the front as something else than what it is in terms of mathematics. Also perspective distortion is inadequately corrected by the eye (especially in large buildings).

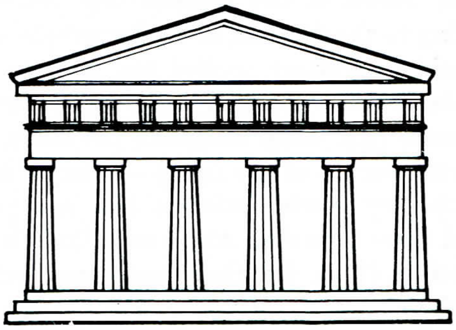


Fig. 11

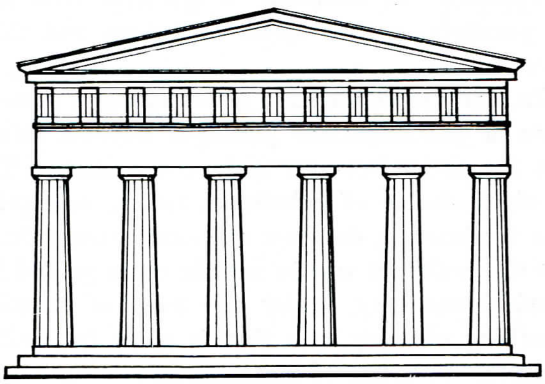


Fig. 12

Our illustrations show how deceptive mathematical truthes are: however similar the scales on the fronts may be, great differences in actual measurements (which may be in the same ratio) though, produce differences in aesthetic effect.

The following method will be adopted:

On the basis of statistical generalization, an "ideal frontal model" will be provided for the Doric temples of the archaic Balkans and South Italy (Figs 11. and 12.), the classical Balkans and South Italy (Figs 14. and 16.), and of the Hellenistic Balkans (Fig. 17.), with the frontal schemes of one of the Sicilian giants, the temple of Apollo

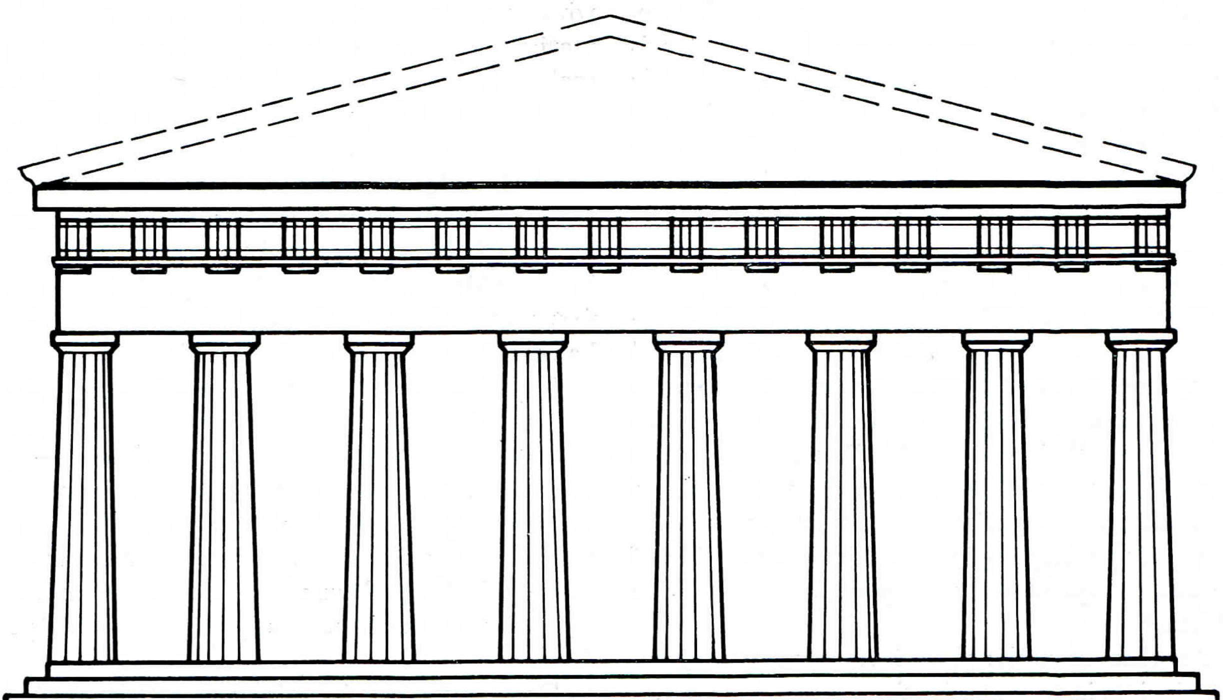


Fig. 13

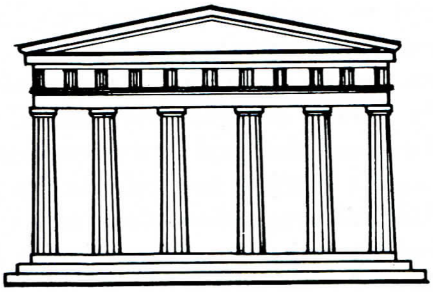


Fig. 14

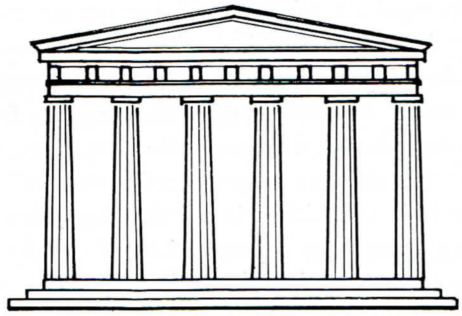


Fig. 17

at Selinus (Fig. 13.) and of the Parthenon (Fig. 15.) fitted in in chronological order.

The numerical figures are given in the following pattern: width in meters (1.), the ratios of width: height of order (2.), height of order : height of column (3.), height of column : diameter of column (4.), intercolumnium : column diameter (5.), and width : height of ridge (6.).

	1.	2.	3.	4.	5.	6.
Arch. Balkans	17.88	1.92	1.57	4.39	1.49	1.40
Arch. S. Italy	20.93	1.72	1.70	4.20	1.29	1.40
Selinus, 'G'	50.07	2.36	1.45	4.95	1.20	1.80 (?)
Cl. Balkans	15.85	1.76	1.36	5.38	1.43	1.47
Parthenon	30.88	2.25	1.32	5.48	1.25	1.72
Cl. S. Italy	20.89	1.81	1.43	4.51	1.18	1.49
Hell. Balkans	16.46	1.62	1.19	6.35	1.35	1.32

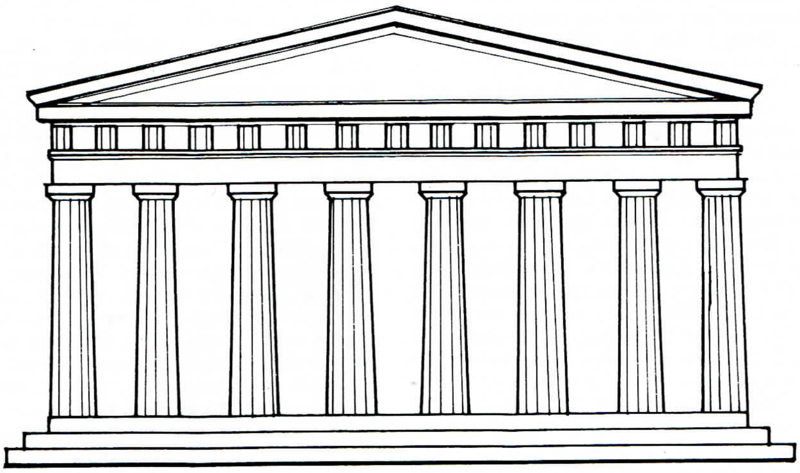


Fig. 15

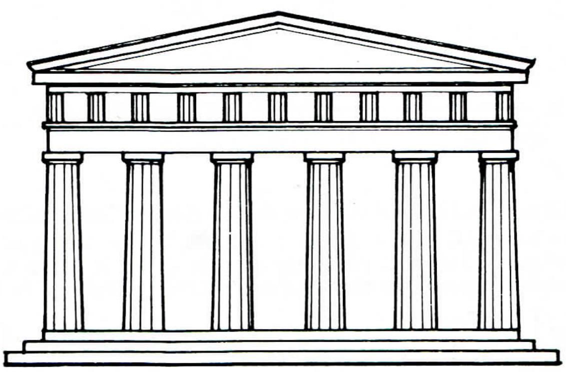


Fig. 16

The differences in measurements and scales are clearly seen in the table. Even more than mathematical figures, illustrative pictures make us realize the different visual effects created even by similar ratios in buildings of differing measurements (Figs 11—17, *with identical scale*).

In preceding chapters an analysis of the main components was given, in accordance with the topic under discussion, which pointed, in most cases, to a mathematical consciousness in planning. In spite of the inevitable distortion resulting from scaling down and schematic simplification, the illustrations of fronts demonstrate now the variety in the realization of an almost universal principle of planning. Even more impressive an effect could be produced by showing the fronts of all the temples on a larger scale than here, though this would still be a poor reflection of reality.

For it is the vision itself that the viewer of a building accepts or rejects without clearly understanding or knowing (especially in the case of modern structures built on higher mathematical principles) the underlying guiding principles and methods of design. Even the spectator of Greek temples based on simple scales cannot accurately percieve dimentional relations and the conceptions of design and planning in the background: to him it is totally immaterial that the ratio between the flank and the width of the Parthenon is 9:4 just as between the axial spacing and column diameters, not to speak of constructions based on more complex ratios or proportions.

There are things which the viewer recognizes instantly (above all symmetry defined by the vertical axis and the equal intercolumniums of the middle on the front). Other scales he can perceive by shifting his eyes several times and by means of optical correction (e.g. the ratio 4:3 between the height of order and height of column of the Parthenon, or perhaps even the ratio 3:1 between the width and height of columns of this building) — but in fact he perceives and admires the total picture, where the building as a whole, together with the elaborate design of its parts, embodies the harmony radiated by rational beauty and disciplined imagination.

NOTES

[1] The data concerning the first four columns are given on the basis of *W. B. Dinsmoor*'s work, the best handbook on the subject to date. Accuracy ranks among its chief merits. Although recent research has proved some of its findings inadequate, we do not know of any other book of comparable value. It is regrettable though that its otherwise comprehensive documentation is not complete: e.g. data concerning the temple of Artemis at Corcyra (Corfu) are missing, and what is even stranger, it fails to provide data for the heights of ridges. This latter deficiency was not Dinsmoor's own fault: we have not found a handbook or even a specialized monograph giving this measurement, a fact that is all the more puzzling since the total height of a building is one of the aesthetically most important factors (at any rate far more important than a host of lesser components so meticulously scrutinized by archeologists). We have thus been compelled to recover it (fifth column) from the reconstructed pictures in titles listed in the bibliography and by means of photogrammetry. The sign — signifies data that cannot be reconstructed, the sign ? data whose value is uncertain or naccessible. A certain degree on uncertainty is always present, since often the principal elements (front and back) display significant differences. — The cultic role of certain temples has not yet been cleared up beyond doubt: these will be marked by signs generally used in literature. The table presents the buildings in chronological order.

[2] On philological issues, R. Falus: Sur la théorie de module de Vitruve. Acta Arch. at press.

[3] Calculations made on a TPAI type computer, manufactured by KFKI.

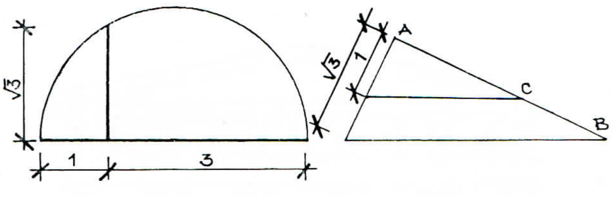
[4] On account of its uncertainty, D. S. Robertson's assumption that the Attic foot was, up to the Roman age, 328 mm (p. 82, note 3) is unacceptable. We did not find this measure characteristic of the main components of buildings. — C. A. Doxiadis writes that its value was 308 mm in pre-Periclean ages, however, this measure ranks only sixth in the most frequent common divisors.

[5] We examine common divisors to be found at least in four main components. The reason why data for buildings reconstructed by three main components (flank, width, column) have been omitted both from here and from the second part of the Appendix is that it is mathematically inevitable that these will have far more common divisors, but it is not certain at all that any of them goes into e.g. the height of order. On the other hand, we did not treat values that can be gauged in four main components separately from those gauged in five, since we know the height of ridge in the case of only 17 buildings.

[6] On the contrary, attributing netted structure to the Greeks, e.g. T. Brunés.

[7] Let us give an illustration. If tolerance values are 80 mm on the flank, 50 mm on the width, 20 mm on the columns, and 50 mm on the height of order (i.e. tolerance is reduced almost by 40% as compared with our previous analysis), we find e.g. that on the temple of Poseidon at Paestum even these four components are without a common divisor. It is also obvious that greater tolerance makes for a larger number of common divisors. Furthermore, it is natural that in large buildings (e.g. the flank of the temple of Zeus Olympius at Acragas or that of Apollo's at Selinus, both measuring over 110 metres), deviation from the plan was significantly greater than in the case of the Hephaesteum or the temple of Poseidon at Sunion, both less than 1:3 of the former two in their respective measurements. It follows that adherence to tolerance of identical strictness inevitably implies the possibility of a certain degree of error (something we shall have to revert to later on), yet it does not weaken our critical statements: reduction in tolerance would be unrealistic — cf. the example of Paestum —, augmentation on the other hand would render the module theory even less tenable.

[8] Supposing that the ratio between width and height of order was planned to be $\sqrt{3}$ (as in the case of the temple of Hera at Selinus), the method of construction may have been the following:



AB is the width scaled down; for the sides of similar triangles we have $AC : AB = 1 : \sqrt{3}$, so AC will be the height of the order. — This was relatively easy to draw; it is, however, totally improbable that the corresponding scale of Selinus 'C' $\left(= \frac{\sqrt{5}+\sqrt{2}}{2}\right)$ was planned in a similar fashion around the middle of the 6th c.: it is doubtful whether the method of extracting roots and formulae of similar triangles were known then, and it is also obvious that the ratio thus obtained was too difficult to draw accurately on the plan, not to mention the task of enlarging it, accurately to the millimetre, on the building itself.

[9] Views of this kind, products of the 19th century, were rejected already by W. B. Dinsmoor, nevertheless they have cropped up again since the 1920s. The theory related to the pentagon drawn into a circle and that of the "golden section" came into fashion chiefly in the wake of E. Moessel's treatise, which — although it provided scanty, accidental and inaccurate evidence — had such an effect with its dazzling speculations otherwise far removed from ancient mathematics that advocates of the "golden section" believed they had found in it the historical justification of their favourite theory. — Basically accepting Vitruvius' theory, and "reconstructing" in a speculative way the design of fronts according to different modules, K. F. Wieninger thought of the hexagon as the basis of ratios. A combination of the two, by no means novel theories and a perfect example of numerical mysticism (through unconditional acceptance of the golden section) is O. Schubert's otherwise reliable and exhaustive book.

[10] Documentation in a spectacular manner is not among our endeavours, and an exhaustive scientific-historical survey would be the job of specialists. Therefore, we only mention that the foot or the ell was attributed the role of module as early as the turn of the century; later authors adopted similar views (e.g. D. S. Robertson and A. W. Lawrence), as well. G. Gruben and several other researchers think axial spacing on the front served as a module for main components. Though this can be proved in some cases (naturally, according to the axial spacing of middle columns), it cannot be generalized on account of a special feature of classic temples (external columns being often contracted). It is M. Borissavliévitch's merit that the reign of the theory of the golden section came to an end, though his assumption that ancient Greeks designed their temples along the laws of perspective is, owing to the scarcity of data, unacceptable. — These works, in addition to others already mentioned, contain those views which we classified into trends at the beginning of this paper. These views are adopted in a variety of ways by the authors of titles in our bibliography.

[11] When saying that a ratio is characteristic of a group, we require two criteria: it should occur at least three times and in more than 1:3 of temples belonging to the group. Without such strictness of standards no certainly or even probably valid generalizations can be made, even though the scarcity of material is bound to beg for a more liberal treatment. In a similar fashion, when the ratio between width and height of order in temples Nos 10. and 19. in group I. is exactly 2:1, and the respective ratios of the others cannot be reconstructed, and 3 out of 5 temples have a ratio width : height of column exactly 3:1, then we could easily declare the said 2:1 ratio characteristic. — Similarly, in the case of temples in group V. We know the ratio width : height of order with three and that of height of order : height of column with again three. And as two-two of these ratios are in each group identical (in the first case Nos 12. and 30. have the ratio 3:1, in the second Nos 11. and 12. have 3:2), we could regard them as a characteristic of 66.7%. However, we think that proper analysis must not be based on so few — therefore accidental — instances of coincidence.

[12] This applies even more to the relationship between front and height of ridge, and between height of ridge and the other two vertical elements (height of order and height of columns). So much so that we might regard it as conclusive evidence were it not for the fact that we have to be satisfied with information only about 17 buildings (circ. 38% of the total number). — Ratios obtained through calculations involving axial spacing (either as dividend or as divisor) are far less accurate than other scales (cf. note 10).

[13] In this chapter we have set higher standards of accuracy: a "good ratio" has to be within 0.03 of the ideal value.

[14] On philological issues, R. Falus: La terminologie grecque du "rapport" et de la "proportion". Acta Ant. at press.

[15] Already Vitruvius called attention to the horizontal curvature, but he did not mention the design of the *scamilli impares* according to fixed ratios. Although theoretically fascinating, W. B. Dinsmoor's formula concerning the curvature on the stylobate of the Hephaesteum is not supported by data (p. 167 sq.). It is even less probable that a formula valid in a particular instance can be extended for general application, whether in respect of horizontal or vertical curves. For the latter see: O. Schubert (135. sq.). Concerning measurements of tympana of the archaic and classic ages see: E. Lapalus (p. 234 sq.); no consistent principle of relating vertical and horizontal measurements can be found.

[16] Koldewey's ingenious and oft-quoted formula on contraction of external columns $\left(\frac{a-t}{2}\right)$, where a is the width of the architrave and t is that of the triglyph, can only be demonstrated on a few temples (e.g. not on the Parthenon). We do not think that values of contraction can be reduced to any mathematical formula and it is out of the question that it should have spread over the Balkans and, later, in South Italy with for the sake of optical correction (external columns seem to be closer to one another even without being contracted). More probably there were statical reasons for contraction.

Appendix

I An arithmetical approximation of ratios of measurements

II Common divisors of main components in millimetres

III Bibliography

I. An arithmetical approximation of ratios of measurements

	Flank : width	Width : height of order	Width : height of columns	Height of order : height of columns	Width : height of ridge	Height of ridge : height of order	Height of ridge : height of columns
1.	2.6672 ≈ 2.6666 8:3 ≈ 8.00:3.00	—	3.5920 ≈ 3.5918 7:2 ≈ 7.04:1.96	—	—	—	—
2.	2.6898 ≈ 2.6846 8:3 ≈ 8.00:2.98	—	2.5777 ≈ 2.5765 18:7 ≈ 18.01:6.99	—	—	—	—
3.	2.7701 ≈ 2.7708 1:4 ≈ 11.00:3.97	—	2.8000 = 2.8000 14:5 ≈ 14.00:5.00	—	—	—	—
4.	2.6631 ≈ 2.6633 8:3 ≈ 7.99:3.00	1.8219 ≈ 1.8215 20:11 ≈ 20.00:10.98 $\sqrt{5}:2 + \sqrt{2}:2 = 1.8225$	2.7652 ≈ 2.7644 11:4 ≈ 11.03:3.99	1.5177 ≈ 1.5176 3:2 ≈ 3.02:1.99	1.3766 ≈ 1.3762 7:5 ≈ 6.95:5.05	1.3240 ≈ 1.3223 4:3 ≈ 3.98:3.01	2.0095 ≈ 2.0100 2:1 ≈ 2.01:1.00
5.	2.1604 ≈ 2.1611 13:6 ≈ 13.01:6.02	2.0632 ≈ 2.0612 2:1 ≈ 2.02:0.98	2.9351 ≈ 2.9314 3:1 ≈ 2.99:1.02	1.4226 ≈ 1.4225 10:7 ≈ 10.00:7.03	1.5300 ≈ 1.5303 3:2 ≈ 3.03:1.98	1.3485 ≈ 1.3490 4:3 ≈ 4.02:2.98	1.9184 ≈ 1.9126 2:1 ≈ 1.97:1.03
6.	2.5053 ≈ 2.5050 5:2 ≈ 5.01:2.00	—	2.9674 ≈ 2.9703 3:1 ≈ 3.00:1.01	—	—	—	—
7.	2.3567 ≈ 2.3569 7:3 ≈ 7.00:2.97	1.9266 ≈ 1.9223 2:1 ≈ 1.98:1.03	2.8431 ≈ 2.8428 17:6 ≈ 17.00:5.98	1.4757 ≈ 1.4729 3:2 ≈ 2.99:2.03	—	—	—
8.	2.2142 ≈ 2.2148 11:5 ≈ 11.03:4.98	—	3.8028 ≈ 3.8020 19:5 ≈ 19.01:5.00	—	—	—	—
9.	2.5392 ≈ 2.5404 5:2 ≈ 5.03:1.98 $\sqrt{2} + \sqrt{5}:2 = 2.5322$	1.8653 ≈ 1.8667 2:1 ≈ 1.96:1.05 $1 + \sqrt{3}:2 = 1.8660$	2.6905 ≈ 2.6913 8:3 ≈ 8.02:2.98	1.4341 ≈ 1.4335 10:7 ≈ 10.02:6.99	1.3866 ≈ 1.3877 7:5 ≈ 6.98:5.03	1.3452 ≈ 1.3456 4:3 ≈ 4.01:2.98	1.9292 ≈ 1.9314 2:1 ≈ 1.97:1.02
10.	2.6450 ≈ 2.0505 2:1 ≈ 2.03:0.99	1.8686 ≈ 1.8667 2:1 ≈ 1.96:1.05 $1 + \sqrt{3}:2 = 1.8660$	2.8784 ≈ 2.3738 3:1 ≈ 2.96:1.03	1.5404 ≈ 1.5408 3:2 ≈ 3.02:1.96	—	—	—
11.	2.1993 ≈ 2.1980 11:5 ≈ 10.99:5.00	2.3562 ≈ 2.3557 7:3 ≈ 7.02:2.98	3.4061 ≈ 3.4068 17:5 ≈ 17.00:4.99	1.4456 ≈ 1.4439 3:2 ≈ 2.96:2.05	—	—	—
12.	2.0875 ≈ 2.0816 2:1 ≈ 2.04:0.98	2.1249 ≈ 2.1250 2:1 ≈ 2.04:0.96 $3\sqrt{2}:2 = 2.1213$	3.0547 ≈ 3.0505 3:1 ≈ 3.02:0.99	1.4376 ≈ 1.4390 3:2 ≈ 2.95:2.05	1.5812 ≈ 1.5813 8:5 ≈ 7.97:5.04	1.3439 ≈ 1.3445 4:3 ≈ 4.02:2.99	1.8925 ≈ 1.8932 2:1 ≈ 1.95:1.03
13.	2.2612 ≈ 2.2610 16:7 ≈ 15.94:7.05	1.6561 ≈ 1.6556 5:3 ≈ 5.00:3.02	2.3733 ≈ 2.3737 7:3 ≈ 7.05:2.97	1.4330 ≈ 1.4327 10:7 ≈ 10.00:6.98	1.4270 ≈ 1.4271 10:7 ≈ 9.99:7.00	1.1606 ≈ 1.1611 7:6 ≈ 6.99:6.02	1.6631 ≈ 1.6633 5:3 ≈ 4.99:3.00
14.	1.4631 ≈ 1.4635 3:2 ≈ 3.00:2.05	1.1328 ≈ 1.1325 8:7 ≈ 7.95:7.02	1.6039 ≈ 1.6040 8:5 ≈ 8.03:5.00	1.4159 ≈ 1.4161 10:7 ≈ 9.97:7.04	1.0572 ≈ 1.0619 1:1 ≈ 1.03:0.97	1.2185 ≈ 1.2177 6:5 ≈ 6.04:4.96	1.6957 ≈ 1.6970 5:3 ≈ 5.04:2.97
15.	2.0834 ≈ 2.0816 2:1 ≈ 2.04:0.98	—	3.1276 ≈ 3.1237 3:1 ≈ 3.02:0.97	$\sqrt{2}$ ≈ 1.4142	—	—	—
16.	2.6515 ≈ 2.6512 8:3 ≈ 7.98:3.01	1.8348 ≈ 1.8350 11:6 ≈ 11.01:6.00	2.5108 ≈ 2.5100 5:2 ≈ 5.02:2.00	1.3684 ≈ 1.3695 4:3 ≈ 4.4 2.95	—	—	—
17.	2.0728 ≈ 2.0714 2:1 ≈ 2.03:0.98	—	2.8804 ≈ 2.8835 3:1 ≈ 2.97:1.03	$0.5 + \sqrt{3}:2$ ≈ 1.3660	—	—	—
18.	2.3124 ≈ 2.3113 7:3 ≈ 6.98:3.02	—	—	—	—	—	—

	Flank : width	Width : height of order	Widht : height of columns	Height of order : height of columns	Widht : height of ridge	Height of ridge : height of order	Height of ridge : height of columns
19.	2.0926 ≈ 2.0928 3:1 ≈ 2.03:0.97	1.9025 ≈ 1.9029 2:1 ≈ 1.96:103	2.6119 ≈ 2.6124 13:5 ≈ 13:01:4.98	1.3729 = 1.3729 4:3 ≈ 4.05:2.95	1.4051 ≈ 1.4048 7:5 ≈ 7.01:4.99	1.3540 ≈ 1.3523 4:3 ≈ 4.03:2.98	1.8589 ≈ 1.8586 13:7 ≈ 13.01:7.00
20.	—	—	—	—			
21.	2.5009 ≈ 2.5000 5:2 ≈ 5.00:2.00	1.7446 ≈ 1.7456 7:4 ≈ 7.00:4.01	2.5258 ≈ 2.5276 5:2 ≈ 5.03:1.99	1.4478 ≈ 1.4461 3:2 ≈ 2.95:2.04			
22.	2.4919 ≈ 2.4900 5:2 ≈ 4.98:2.00	—	—	—			
23.	2.6747 ≈ 2.6756 8:3 ≈ 8.00:2.99	1.7321 ≈ 1.7320 7:4 ≈ 5.68:4.03 $\sqrt{3} = 1.7321$	2.4950 ≈ 2.4950 5:2 ≈ 4.99:2.00	1.4404 ≈ 1.4390 3:2 ≈ 2.95:2.05			
24.	2.3165 ≈ 2.3156 7:3 ≈ 6.97:3.01	1.9076 ≈ 1.9038 2:1 ≈ 1.98:1.04	2.6539 ≈ 2.6545 8:3 ≈ 7.99:3.01	1.3912 ≈ 1.3904 7:5 ≈ 6.98:5.02	1.4802 = 1.4802 3:2 ≈ 2.99:2.02	1.2888 ≈ 1.2890 9:7 ≈ 9.01:6.99	1.7929 ≈ 1.7928 9:5 ≈ 9.00:5.02
25.	2.4718 ≈ 2.4703 5:2 ≈ 4.99:2.02	1.9169 ≈ 1.9135 2:1 ≈ 1.99:1.04 $0.5\sqrt{2} = 1.9142$	2.7324 ≈ 2.7313 11:4 ≈ 10.98:4.02 $1 + \sqrt{3} = 2.7321$	1.4255 ≈ 1.4251 10:7 ≈ 9.99:7.01	1.5974 ≈ 1.5980 8:5 ≈ 7.99:5.00	1.1991 ≈ 1.1980 6:5 ≈ 5.99:5.00	1.7106 ≈ 1.7114 12:7 ≈ 11.98:7.00
26.	2.2531 ≈ 2.2525 9:4 ≈ 9.01:6.00	1.8261 ≈ 1.8263 9:5 ≈ 9.04:495	2.6588 ≈ 2.6578 8:3 ≈ 8.00:3.01	1.4514 ≈ 1.4510 3:2 ≈ 2.96:2.04			
27.	2.4988 ≈ 2.4950 5:2 ≈ 4.99:2.00	1.7891 ≈ 1.7888 9:5 ≈ 8.98:5.02	2.5868 ≈ 2.5846 5:2 ≈ 5.04:1.95	1.4459 ≈ 1.4461 3:2 ≈ 2.95:2.04			
28.	2.6425 ≈ 2.6424 8:3 ≈ 7.98:3.02	1.8315 ≈ 1.8311 20:11 ≈ 20.05:10.95	2.4304 ≈ 2.4314 5:2 ≈ 4.96:2.04	1.3270 ≈ 1.3256 4:3 ≈ 3.99:3.01			
29.	2.3176 ≈ 2.3179 7:3 ≈ 7.00:3.02	1.7727 ≈ 1.7723 9:5 ≈ 8.95:5.05	2.3994 ≈ 2.4000 12:5 ≈ 12.00:5.00	1.3536 ≈ 1.3535 4:3 ≈ 4.02:2.97	1.4532 = 1.4532 3:2 ≈ 2.95:2.03	1.2198 = 1.2198 6:5 ≈ 6.05:4.96	1.6511 ≈ 1.6512 5:3 ≈ 4.97:3.01
30.	2.2507 ≈ 2.2500 9:4 ≈ 9.00:4.00	2.2494 ≈ 2.2500 9:4 ≈ 9.00:4.00	2.9598 ≈ 2.9604 3:1 ≈ 2.99:1.01	1.3158 ≈ 1.3146 4:3 ≈ 3.97:3.02	1.7194 ≈ 1.7203 7:4 ≈ 6.95:4.04	1.3083 ≈ 1.3092 4:3 ≈ 3.98:3.04	1.7215 ≈ 1.7228 7:4 ≈ 6.96:4.04
31.	2.3105 ≈ 2.3103 7:3 ≈ 7.00:3.03	1.6766 ≈ 1.6756 5:3 ≈ 5.01:2.99	2.2361 ≈ 2.2363 9:4 ≈ 8.99:4.02 $\sqrt{5} = 2.2361$	1.3334 ≈ 1.3333 4:3 ≈ 4.00:3.00	—	—	—
32.	2.3127 ≈ 2.3146 7:3 ≈ 6.99:3.02	1.7278 ≈ 1.7277 7:4 ≈ 6.98:4.04	2.2859 ≈ 2.2854 9:4 ≈ 9.05:3.96 $\sqrt{2} + \sqrt{3}:2 = 2.2802$	1.3230 ≈ 1.3234 4:3 ≈ 4.01:3.03	—	—	—
33.	—	—	—	—	—	—	—
34.	2.1429 ≈ 2.1422 13:6 ≈ 12.96:6.05	1.8194 ≈ 1.8198 20:11 ≈ 20.00:10.99	2.4380 ≈ 2.4363 5:2 ≈ 4.97:2.04 $\sqrt{3} + \sqrt{2}:2 = 2.4392$	1.3400 = 1.3400 4:3 ≈ 4.02:3.00	—	—	—

	Flank : width	Width : height of order	Widht : height of columns	Height of order : height of columns	Widht : height of ridge	Height of ridge : height of order	Height of ridge : height of columns
35.	2.3291 ≈ 2.3289 7:3 ≈ 7.01:3.01	1.7165 ≈ 1.7160 7:4 ≈ 6.95:4.05	2.5261 ≈ 1.5253 5:2 ≈ 5.00:1.98	1.4716 ≈ 1.4703 3:2 ≈ 2.97:2.02	1.4223 = 1.4223 10:7 ≈ 9.97:7.01	1.2319 ≈ 1.2333 5:4 ≈ 4.97:4.03	1.7761 ≈ 1.7762 9:5 ≈ 8.97:5.05
36.	1.7566 ≈ 1.7569 7:4 ≈ 7.01:3.99	1.5811 ≈ 1.5813 8:5 ≈ 7.97:5.04	2.0830 ≈ 2.0825 2:1 ≈ 2.02:0.97	1.3174 ≈ 1.3179 4:3 ≈ 3.98:3.02	1.2690 ≈ 1.2695 5:4 ≈ 5.04:3.97	1.2460 ≈ 1.2475 5:4 ≈ 4.99:4.00	1.6415 ≈ 1.6403 5:3 ≈ 4.97:3.03
37.	2.5102 ≈ 2.5100 5:2 ≈ 5.02:2.00	1.7852 ≈ 1.7853 9:5 ≈ 8.98:5.03	2.4685 ≈ 2.4653 5:2 ≈ 4.98:2.02	1.3828 ≈ 1.3829 7:5 ≈ 6.97:5.04	1.4547 ≈ 1.4559 3:2 ≈ 2.97:2.04	1.2275 ≈ 1.2277 5:4 ≈ 4.96:4.04	1.6973 ≈ 1.6970 5:3 ≈ 5.04:2.97
38.	2.1323 ≈ 2.1325 15:7 ≈ 14.97:7.02	1.7515 ≈ 1.7525 7:4 ≈ 7.01:4.00	2.3385 ≈ 2.3389 7:3 ≈ 7.04:3.01	1.3351 ≈ 1.3355 4:3 ≈ 4.02:3.01	—	—	—
39.	1.9609 ≈ 1.9604 2:1 ≈ 1.98:1.01	1.7500 ≈ 1.7500 7:4 = 7.00:4.00	2.2615 ≈ 2.2607 9:4 ≈ 9.02:3.99	1.2923 = 1.2923 9:7 ≈ 9.02:6.98	1.2653 ≈ 1.2663 5:4 ≈ 5.04:3.98	1.3125 ≈ 1.3135 4:3 ≈ 3.98:3.03	1.6962 ≈ 1.6970 5:3 ≈ 5.04:2.97
40.	2.6836 ≈ 2.6846 8:3 ≈ 8.00:2.98	—	—	—	—	—	—
41.	2.4779 ≈ 2.4776 5:2 ≈ 4.98:2.01	1.6133 ≈ 1.6137 8:5 ≈ 8.02:4.97 0.5 + √5:2 = 1.6180	2.0255 ≈ 2.0202 2:1 ≈ 2.00:0.99	1.2555 ≈ 1.2550 5:4 ≈ 5.02:4.00	1.3726 ≈ 1.3725 11:8 ≈ 10.98:8.00	1.1753 ≈ 1.1739 7:6 ≈ 7.02:5.98	1.4756 ≈ 1.4752 3:2 ≈ 2.98:2.02
42.	2.1182 ≈ 2.1158 2:1 ≈ 2.01:0.95 1 + √5:2 = 2.1180	1.5532 ≈ 1.5538 3:2 ≈ 3.03:1.95	1.9377 ≈ 1.9320 2:1 ≈ 1.99:1.03	1.2476 ≈ 1.2475 5:4 ≈ 4.99:4.00	—	—	—
43.	1.9565 ≈ 1.9510 2:1 ≈ 1.99:1.02	1.8078 ≈ 1.8076 9:5 ≈ 9.02:4.99	2.3354 ≈ 2.3367 7:3 ≈ 7.01:3.00	1.2919 ≈ 1.2923 9:7 ≈ 9.02:6.98	—	—	—
44.	1.9463 ≈ 1.9412 2:1 ≈ 1.98:1.02	—	—	—	—	—	—
45.	1.3718 ≈ 1.3729 4:3 ≈ 4.05:2.95	1.6890 ≈ 1.6879 5:3 ≈ 5.03:2.98	2.1746 ≈ 2.1753 13:6 ≈ 13.03:5.99	1.2875 ≈ 1.2871 9:7 ≈ 9.01:7.00	—	—	—
46.	2.2879 ≈ 2.2895 7:3 ≈ 6.96:3.04	1.7176 ≈ 1.7185 7:4 ≈ 6.96:4.05	2.3981 ≈ 2.3972 12:5 ≈ 12.01:5.01	1.3962 ≈ 1.3964 7:5 ≈ 7.01:5.02	—	—	—
47.	1.7742 ≈ 1.7733 7:4 ≈ 7.04:3.97	1.8921 ≈ 1.8942 2:1 ≈ 1.97:1.04	2.3327 ≈ 2.3322 7:3 ≈ 7.02:3.01	1.2329 ≈ 1.2333 5:4 ≈ 4.97:4.03	1.4436 ≈ 1.4439 3:2 ≈ 2.96:2.05	1.3061 ≈ 1.3059 4:3 ≈ 3.97:3.04	1.6103 ≈ 1.6104 8:5 ≈ 8.02:4.98
48.	1.4982 ≈ 1.5000 3:2 = 3.00:2.00	1.2669 ≈ 1.2663 5:4 ≈ 5.04:3.98	1.5067 ≈ 1.5075 3:2 ≈ 3.00:1.99	1.1893 ≈ 1.1892 6:5 ≈ 5.97:5.02	—	—	—
49.	1.9146 ≈ 1.9126 2:1 ≈ 1.97:1.03 0.5 + √2 = 1.9142	1.0825 ≈ 1.0833 1:1 ≈ 1.04:0.96	1.4216 ≈ 1.4217 10:7 ≈ 9.98:7.02	1.3132 ≈ 1.3135 4:3 ≈ 3.98:3.03	—	—	—

II. Common divisors of main components in millimetres (No. of temples with five measured components are in italics)

No.	Common divisors
4.	725, 507, 413, 412, 411, 375, 348, 347, 346, 322, 320, 300, 299, 298
5.	482, 481, 402, 401, 400, 399, 398, 344, 343, 342, 341, 340, 321, 320, 299, 298, 297
7.	641, 491, 439, 438, 437, 436, 395, 394, 363, 362, 348, 334, 333, 332, 308, 307, 306, 298
9.	763, 569, 568, 567, 506, 436, 398, 365, 364, 352, 326, 325, 313, 305, 304
10.	817, 573, 496, 495, 494, 436, 435, 410 368, 354, 338, 337, 328, 310, 309, 308
11.	735, 667, 641, 589, 588, 545, 506, 473, 460, 432, 407, 388, 368, 343, 327, 313, 307, 300, 294
12.	556, 508, 507, 412, 393, 376, 359, 352, 338, 326, 325, 320, 309, 308, 298, 293
13.	441, 440, 439, 438, 381, 363, 362, 340, 339, 338, 325, 324, 323, 322, 305, 304
14.	594, 593, 417, 416, 415, 414, 413, 411, 371, 348, 347, 346, 345, 344, 320, 297
16.	722, 721, 528, 460, 459, 458, 420, 373, 372, 361, 346, 337, 336, 315, 305, 295
19.	659, 658, 657, 656, 655, 654, 653, 407, 406, 405, 404, 380, 330, 329, 328, 327, 313, 312, 293, 292, 291
21.	788, 394, 380, 362, 361, 350, 349, 348, 334, 324, 323, 302, 301, 300
23.	634, 633, 632, 564, 563, 506, 443, 407, 377, 338, 317, 316, 309, 299, 298, 291
24.	521, 455, 454, 374, 373, 347, 338, 315, 308, 307, 298, 297, 291
25.	1275, 1274, 1273, 1272, 637, 636, 425, 424, 372, 318, 316, 307, 297, 296, 295
26.	708, 707, 706, 705, 580, 425, 424, 423, 422, 374, 354, 353, 333, 320, 319, 318, 301, 291, 290
27.	897, 896, 895, 894, 893, 892, 698, 697, 476, 448, 447, 446, 414, 413, 392, 391, 349, 345, 344, 312, 311, 310, 299, 298
28.	662, 661, 660, 659, 658, 657, 499, 498, 497, 496, 400, 373, 372, 331, 330, 329, 316, 315, 314, 313, 296
29.	406, 405, 383, 382, 354, 336, 335, 334, 319, 300, 299
30.	476, 475, 474, 418, 417, 416, 402, 401, 373, 372, 360, 359, 335, 326, 325, 306, 299, 297, 291
31.	674, 673, 502, 501, 500, 499, 467, 399, 353, 352, 337, 336, 335, 319, 301, 300, 299
32.	420, 419, 350, 349, 348, 332, 300, 299, 298, 297
34.	458, 457, 456, 455, 454, 453, 452, 372, 371, 370, 369, 345, 344, 343, 342, 294, 293, 292
35.	514, 446, 445, 444, 395, 394, 393, 392, 354, 353, 352, 351, 337, 321, 320, 319, 318, 306, 292, 291
36.	515, 514, 513, 512, 388, 387, 386, 385, 384, 361, 360, 359, 358, 357, 335, 310, 293, 292, 291
37.	724, 723, 445, 444, 407, 406, 391, 362, 361, 360, 334, 322, 309, 303, 293
38.	2473, 2472, 2471, 2470, 2469, 2468, 2467, 2465, 2464, 2463, 2462, 2461, 2460, 2459, 2458, 1236, 1235, 1234, 1233, 1232, 1231, 1230, 1229, 824, 823, 822, 821, 820, 618, 617, 616, 615, 496, 494, 493, 492, 413, 412, 411, 410, 392, 369, 368, 354, 353, 352, 338, 321, 320, 310, 309, 308, 297
39.	741, 740, 739, 738, 370, 369, 368, 324, 308, 307, 304, 303
41.	410, 350, 338, 315, 306, 305, 296, 295
42.	647, 646, 645, 517, 372, 371, 359, 358, 357, 347, 334, 325, 324, 323, 315, 314, 296, 295
43.	505, 504, 416, 415, 396, 395, 354, 353, 340, 339, 338, 337, 324, 308, 307, 297, 296, 295
45.	734, 733, 508, 507, 506, 466, 465, 464, 463, 367, 361, 317, 316, 302, 301, 300, 299, 298
46.	652, 522, 521, 520, 519, 518, 517, 516, 432, 431, 430, 403, 402, 401, 400, 349, 348, 347, 346, 345, 344, 328, 327, 326, 306, 305, 304, 303
47.	431, 430, 345, 340, 338, 307, 305, 297, 291
48.	516, 506, 466, 465, 464, 359, 358, 357, 334, 317, 303, 302, 298
49.	521, 518, 517, 516, 432, 431, 346, 337, 315, 309, 303

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INDEX

J. Białostocki: Die Internationalität der Kunstgeschichte und das Comité International d'Histoire de l'Art 175

Éva Moskovszky—Z. Kádár: To the Interpretation of an Early Christian Marble Fragment in the Museum of Fine Arts, Budapest 179

Lívia Varga: Die mittelalterliche Baugeschichte der evangelischen Kirche in Mühlbach . 187

Katalin Granasztói-Győrffy: Nuovi contributi all'attività del pittore veronese Sebastiano Vini 237

S. Kontha: Béni Ferenczy 253

R. Falus—T. Mezős: Scales and Proportions on Doric Buildings 281

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Neue Bücher über den Barock *(Klára Garas)* 319

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